Helmholtz resonator with extended neck\textsuperscript{a)}

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Acoustic performance of a concentric circular Helmholtz resonator with an extended neck is investigated theoretically, numerically, and experimentally. The effect of length and shape of, and the perforations on the neck extension is examined on the resonance frequency and the transmission loss. A two-dimensional analytical method is developed for an extended neck with constant cross-sectional area, while a three-dimensional boundary element method is applied for the variable area and perforated extension. Lumped and one-dimensional approaches are also included to illustrate the effect of the higher order modes. For a piston-driven model, predicted resonance frequencies using lumped, one-dimensional, and two-dimensional analytical methods are compared with those from multidimensional boundary element method. Analytical and computational transmission loss predictions for pipe-mounted model are compared to the experimental data obtained from an impedance tube setup. It is shown that the resonance frequency may be controlled by the length, shape, and perforation porosity of the extended neck without changing the cavity volume. © 2003 Acoustical Society of America. [DOI: 10.1121/1.1558379]

LIST OF SYMBOLS

- \( a_1 \) Neck radius
- \( a_2 \) Cavity radius
- \( A_n \) Modal amplitudes in domain I
- \( B_n \) Modal amplitudes in domain II
- \( C_n \) Modal amplitudes in domain III
- \( c_0 \) Speed of sound
- \( f_r \) Resonance frequency
- \( G \) Green’s function
- \( J_m \) Bessel function of the first kind and order \( m \)
- \( k \) Wave number
- \( k_0 \) Wave number of mode \((0,0)\)
- \( k_n \) Wave number of mode \((0,n)\)
- \( \ell_1 \) Base neck length
- \( \ell_2 \) Neck extension length
- \( \ell_c \) Total neck length
- \( \ell_3 \) Unit normal vector in the outward direction
- \( S \) Cross-sectional area of acoustic domains
- \( S_n \) Neck cross-sectional area
- \( S_p \) Pipe cross-sectional area
- \( T \) Impedance matrix
- \( T_{ij} \) Transfer matrix elements
- \( TL \) Transmission loss
- \( u \) Acoustic velocity
- \( u_p \) Piston velocity
- \( V_c \) Cavity volume
- \( x_1, x_2, x_3 \) Coordinates
- \( x, y, z \) Cartesian coordinates
- \( \beta_n \) Roots of Eq. (9)
- \( \gamma \) Acoustic domain boundary
- \( \delta \) End correction
- \( \rho_0 \) Density of air
- \( \rho \) Duct porosity
- \( \eta \) Eigenfunctions
- \( \omega \) Angular velocity

Superscript

- + Traveling in the positive direction
- \( - \) Traveling in the negative direction
- \( i \) Inlet
- \( o \) Outlet
- \( r \) Rigid wall

Subscript

- \( A \) Domain I
- \( B \) Domain II
- \( C \) Domain III
- \( c \) Cavity
- \( M \) Main duct

I. INTRODUCTION

Helmholtz resonator is an effective acoustic attenuation device at low frequencies with its resonance dictated by the combination of cavity and neck and their relative orientation. The classical lumped analysis of this attenuator gives the resonance frequency as

\[ f_r = \frac{c_0}{2\pi} \sqrt{S_n [V_c (\ell_n + \delta_n)]} \]

where \( c_0 \) is speed of sound, \( S_n \) the neck cross-sectional area, \( V_c \) the resonator volume, \( \ell_n \) the neck length, and \( \delta_n \) the end correction to account for higher modes excited at the discontinuities, which can be determined by the geometry and

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Ingard\textsuperscript{1,2} investigated the effect of neck geometry, such as cross-sectional area shape, location, and size, on the resonance frequency of a Helmholtz resonator with circular or rectangular cross-sectional area for volume. He developed end corrections for both single and double holes to account for the higher order modes at the interface between neck and cavity. Chanaud\textsuperscript{3} examined the effect of both orifice and cavity geometry on the resonance frequency of a Helmholtz resonator using Ingard’s end correction. The effect of the depth and width for variable and fixed volumes was studied. He presented the limitations of simple lumped and transcendental models based on the predictions, and concluded that for a fixed volume and orifice size, the orifice position changed the resonance frequency substantially, while the orifice shape was not significant. References 1 and 2 considered only very short neck length compared to wavelength. Tang and Sirignano\textsuperscript{3} studied a Helmholtz resonator with a neck comparable to wavelength as an application to reducing combustion instability. They developed a general formulation and applied it to quarter wave resonators and Helmholtz resonators with various neck lengths.

Recently, Selamet and co-workers\textsuperscript{4–6} employed several approaches to examine the effect of cavity volume and neck locations. They illustrated the effect of length-to-diameter ratio of the volume on the resonance frequency and transmission loss characteristics using lumped and one-dimensional (1D) approaches.\textsuperscript{4} Earlier works have been extended further by studying a number of circular concentric configurations with lumped, one-dimensional radial and axial, two-dimensional (2D) analytical approaches, and three-dimensional (3D) boundary element method (BEM).\textsuperscript{5} They also developed a 3D analytical approach to investigate the effect of neck offset on the behavior of circular asymmetric Helmholtz resonator.\textsuperscript{6}

While the Helmholtz resonator is known to be an effective acoustic reflector at low frequencies, the use of it may, at times, be limited due to volume constraints. Thus, it is important to lower the resonance frequency without increasing volume or reduce volume without increasing resonance frequency. While a wealth of literature exists on the effect of neck cross-sectional area and opening location to volume, the impact of neck extension remains to be investigated. The present work therefore concentrates on the effect of neck extension geometry (Fig. 1) on the Helmholtz resonator behavior. The neck extension length will shift the resonance frequency without changing the volume. Such an extension also acts like a quarter wave resonator in the cavity resulting in additional transmission loss peaks at higher frequencies compared to a resonator without extension. Furthermore, the neck shape involving either variable cross-sectional area or perforations can readily change the acoustic characteristics of the resonator. Such an ability to modify the resonance frequency without changing the volume may be desirable in a variety of applications, including adaptive passive-noise control.\textsuperscript{7–9} Among other designs, Cheng and co-workers,\textsuperscript{8} for example, suggested two concentric cylindrical neck extensions with preferably parabolic cut-outs. The rotation of the cylindrical walls varies the alignment of cut-outs, therefore effectively changing the neck length.

The objective of the present study is, in the absence of mean flow, to (1) investigate theoretically, numerically, and experimentally the acoustic attenuation performance of a concentric circular Helmholtz resonator with an extended neck; and (2) examine the effects of length, shape, and perforation of the neck extension on the resonator behavior. A two-dimensional analytical method is developed for an extended neck with constant cross-sectional area, while a three-dimensional boundary element method is applied for the variable-area and perforated extension. Lumped and one-dimensional approaches are also presented to illustrate the effect of the higher order modes. For a piston-driven model, predicted resonance frequencies using lumped, 1D, and 2D analytical methods are compared with those from 3D BEM. Analytical and computational transmission loss predictions for pipe-mounted model are compared to the experimental data obtained from an impedance tube setup.

Following this Introduction, Sec. II develops a two-dimensional analytical approach and Sec. III summarizes a 3D BEM. Section IV compares the analytical and computational predictions with experiments, and discusses the effect of geometry of the neck extension on the resonance frequency and transmission loss characteristics. Section V concludes the study with final remarks.

II. TWO-DIMENSIONAL ANALYTICAL APPROACH

A two-dimensional analytical approach is introduced next to determine the acoustic characteristics of a piston-
driven circular Helmholtz resonator with an extended neck, which consists of circular and annular ducts (Fig. 2). For a two-dimensional axisymmetric wave propagation in a circular or annular duct, the Helmholtz equation is given, in cylindrical coordinates $(r, x)$, by

$$\nabla^2 p(r, x) + k^2 p(r, x) = 0,$$

where $P$ and $k$ are the acoustic pressure and the wave number, respectively. The solution to Eq. (1) is obtained next in three domains: I—neck, II—annular, and III—circular in the volume.

A. Solution for circular neck (domain I)

The solution to Eq. (1) in domain I or circular neck (Fig. 2) can be written as

$$P_A(r, x_1) = \sum_{n=0}^{\infty} (A_n^+ e^{-jk_{A,n}x_1} + A_n^- e^{jk_{A,n}x_1}) \psi_{A,n}(r),$$

where $P_A$ is the acoustic pressure, $A_n^+$ and $A_n^-$ are the modal amplitudes corresponding to components traveling in the positive and negative $x_1$ directions in domain I, respectively, $k_{A,n}$ the wave number, and $\psi_{A,n}(r)$ the eigenfunctions. For this circular duct, the eigenfunctions are given by

$$\psi_{A,n}(r) = J_0 \left( \frac{\alpha_n r}{a_1} \right),$$

where $J_0$ is the Bessel function of the first kind and order $m$, $a_1$ the duct diameter, and $\alpha_n$ the roots satisfying the rigid wall boundary condition of

$$J_0'(\alpha_n) = J_1(\alpha_n) = 0,$$

and

$$k_{A,n} = \begin{cases} \sqrt{\frac{k_0^2 - (\alpha_n/a_1)^2}{a_1}}, & k_0 > \frac{\alpha_n}{a_1} \\ -\sqrt{\frac{k_0^2 - (\alpha_n/a_1)^2}{a_1}}, & k_0 < \frac{\alpha_n}{a_1} \end{cases}$$

the axial wave number of the mode $(0, n)$, and $k_0$ wave number of the mode $(0, 0)$. The negative sign in Eq. (5) is assigned so that $e^{-jk_{A,n}x_1}$ decays exponentially in $x_1$.

The particle velocity may then be written, in terms of the linearized momentum equation, as

$$u_A(r, x_1) = \frac{1}{\rho_0 \omega} \sum_{n=0}^{\infty} k_{A,n} \left[ A_n^+ e^{-jk_{A,n}x_1} - A_n^- e^{jk_{A,n}x_1} \right] \psi_{A,n}(r),$$

where $\rho_0$ is the medium density and $\omega$ the angular velocity.

B. Solution for annular volume (domain II)

For a concentric annular duct with inner and outer duct diameters of $a_1$ and $a_2$, the solution to Eq. (1) can be written in domain II as

$$P_B(r, x_2) = \sum_{n=0}^{\infty} (B_n^+ e^{-jk_{B,n}x_2} + B_n^- e^{jk_{B,n}x_2}) \psi_{B,n}(r),$$

with

$$\psi_{B,n}(r) = J_0 \left( \frac{\beta_n r}{a_2} \right) - \frac{J_1(\beta_n)}{Y_1(\beta_n)} Y_1 \left( \frac{\beta_n r}{a_2} \right),$$

where $Y_m$ is the Bessel function of the second kind and order $m$, $\beta_n$ the root satisfying the boundary condition of

$$J_1 \left( \frac{\beta_n a_1}{a_2} \right) - \frac{J_1(\beta_n)}{Y_1(\beta_n)} Y_1 \left( \frac{\beta_n a_1}{a_2} \right) = 0,$$

and

$$k_{B,n} = \begin{cases} \sqrt{\frac{k_0^2 - (\beta_n/a_2)^2}{a_2}}, & k_0 > \frac{\beta_n}{a_2} \\ -\sqrt{\frac{k_0^2 - (\beta_n/a_2)^2}{a_2}}, & k_0 < \frac{\beta_n}{a_2} \end{cases}$$

Again the negative sign in Eq. (10) is assigned so that $e^{-jk_{B,n}x_2}$ decays exponentially in $x_2$. The particle velocity is then

$$u_B(r, x_2) = \frac{1}{\rho_0 \omega} \sum_{n=0}^{\infty} k_{B,n} \left[ B_n^+ e^{-jk_{B,n}x_2} - B_n^- e^{jk_{B,n}x_2} \right] \psi_{B,n}(r).$$

C. Solution for circular volume (domain III)

The solution to Eq. (1) in domain III or circular volume (Fig. 2) can be written as

$$P_C(r, x_3) = \sum_{n=0}^{\infty} (C_n^+ e^{-jk_{C,n}x_3} + C_n^- e^{jk_{C,n}x_3}) \psi_{C,n}(r)$$

with

$$\psi_{C,n}(r) = J_0 \left( \frac{\gamma_n r}{a_1} \right),$$

where $\gamma_n$ is the root satisfying the boundary condition of

$$J_0 \left( \frac{\gamma_n a_1}{a_1} \right) = 0.$$
\[ \psi_{C,n}(r) = J_0 \left( \frac{\alpha_n r}{a} \right), \]  
where \( \alpha_n \) are the roots satisfying the rigid wall boundary condition of Eq. (4), and
\[ k_{c,n} = \begin{cases} \sqrt{k_0^2 - \left( \frac{\alpha_n}{a} \right)^2}, \ k_0 > \frac{\alpha_n}{a} \\ -\sqrt{k_0^2 - \left( \frac{\alpha_n}{a} \right)^2}, \ k_0 < \frac{\alpha_n}{a}. \end{cases} \]  

The particle velocity is then
\[ u_c(x_3) = \frac{1}{\rho_0 a_n} \sum_{n=-\infty}^{\infty} k_{c,n} \left[ C_+^n e^{-jk_{c,n} x_3} - C_-^n e^{jk_{c,n} x_3} \right] \psi_{C,n}(r). \]  

### D. Boundary conditions

The coefficients in Eqs. (2), (7), and (12) are determined next by the boundary conditions. At \( x_1 = 0 \) and \( x_3 = l_3 \), the rigid boundary conditions, \( u_B = 0 \) and \( u_C = 0 \), give, in view of Eqs. (11) and (15),
\[ B_n^+ = B_n^- \]  
and
\[ C_n^- = C_n^+ e^{-2jk_{c,n}l_3}. \]  

At \( x_1 = \ell_1 \) or \( x_3 = 0 \), the pressure continuity condition between domains I and III, \( P_A = P_C \), results in (Miles\textsuperscript{11} and Selamet and Ji\textsuperscript{12})
\[ (A_1^+ e^{-jk_{A1}l_1} + A_1^- e^{jk_{A1}l_1}) (\psi_{A,1}, \psi_{A,1})_A = \sum_{n=-\infty}^{\infty} (C_n^+ + C_n^-) (\psi_{C,n}, \psi_{A,1})_A. \]  

where \( \langle \cdot \rangle \) indicates the integration over area \( S \) and \( x = 0,1,2,\ldots,\infty \), which is deferred to Appendix A. Similarly, at \( x_2 = \ell_2 \) or \( x_3 = 0 \), the pressure continuity between domains II and III, \( P_B = P_C \), leads to
\[ (B_2^+ e^{-jk_{B2}l_2} + B_2^- e^{jk_{B2}l_2}) (\psi_{B,2}, \psi_{B,2})_B = \sum_{n=-\infty}^{\infty} (C_n^+ + C_n^-) (\psi_{C,n}, \psi_{B,2})_B. \]  

At \( x_1 = \ell_2 \) or \( x_2 = \ell_2 \), the volume velocity continuity, \( u_A S_A + u_B S_B = u_C S_C \), gives
\[ \sum_{n=-\infty}^{\infty} k_{A,n} (A_1^+ e^{-jk_{A,n}l_1} - A_1^- e^{jk_{A,n}l_1}) (\psi_{A,n}, \psi_{C,1})_A + \sum_{n=-\infty}^{\infty} k_{B,n} (B_2^+ e^{-jk_{B,n}l_2} - B_2^- e^{jk_{B,n}l_2}) (\psi_{B,n}, \psi_{C,2})_B = k_{C,n} (C_+^n - C_-^n) (\psi_{C,n}, \psi_{C,n})_C. \]  

### E. Resonance frequency and transmission loss

The expressions for the resonance frequency and transmission loss are given next for the piston-driven and pipe-mounted Helmholtz resonators, respectively. Assuming decay of the higher order modes through neck from \( x_1 = \ell_n \) to \( x_1 = 0 \) (Selamet \textit{et al.}), the piston oscillating with velocity \( u_p \) at \( x_1 = 0 \) gives
\[ \frac{1}{\rho_0 c_0} (A_0^+ - A_0^-) = u_p. \]  

Once Eqs. (16)–(21) are solved using \( s = n \) and letting \( \rho_0 c_0 u_p = 1 \), the acoustic impedance of a Helmholtz resonator at \( x_1 = 0 \) is calculated by
\[ Z_H = \frac{p_A}{\rho_0 c_0 u_p} = A_0^+ + A_0^-. \]  

which then allows the evaluation of resonance frequency.

For a pipe-mounted Helmholtz resonator, transmission loss can be determined, with the assumptions of anechoic termination at the exit of the main duct and plane wave propagation through the main duct, by
\[ TL = 10 \log_{10} \left| 1 + \frac{S_n}{2Sp} \right| \left| \frac{1}{Z_H} \right|, \]  

where \( S_n \) and \( S_p \) are the area of the neck and main pipe, respectively.

### III. THREE-DIMENSIONAL BOUNDARY ELEMENT METHOD

The wave propagation is governed by the Helmholtz equation in Cartesian coordinates,
\[ \nabla^2 P(x,y,z) + k^2 P(x,y,z) = 0, \]  

and the Green’s theorem results in boundary integral equation as follows:
\[ C(q_1) P(q_1) = \int_{\Gamma} \left[ G(q_1, q_2) \frac{\partial P(q_2)}{\partial n} - P(q_2) \frac{\partial G(q_1, q_2)}{\partial n} \right] d\Gamma(q_2), \]  

where \( q_1 \) and \( q_2 \) are points on the boundary surface \( \Gamma \), \( C(q_1) \) coefficient, and \( G(q_1, q_2) \) the Green’s function or fundamental solution given, for a three-dimensional acoustic domain, by
\[ G(q_1, q_2) = \frac{e^{-jk|q_1-q_2|}}{4\pi|q_1-q_2|}. \]  

### A. Piston-driven model

Discretizing the boundary surfaces into a number of elements and applying numerical integration to Eq. (25) yields\textsuperscript{13} for domain I
\[ \begin{pmatrix} P_I(x_1=0) \\ P_I(x_1=\ell_n) \end{pmatrix} = \begin{bmatrix} T_1 & \begin{pmatrix} u_1(x_1=0) \\ u_1(x_1=\ell_n) \end{pmatrix} \end{bmatrix}, \]  

and for the domain II+III,
\[ \begin{pmatrix} P_{II+III}(x_3=0) \\ P^r_{II+III} \end{pmatrix} = \begin{bmatrix} T_2 & \begin{pmatrix} u_{II+III}(x_3=0) \\ u^r_{II+III} \end{pmatrix} \end{bmatrix}, \]
where superscript $r$ indicates rigid wall boundary. Using rigid wall boundary condition, $u_r^i = u_r^{i+1} = 0$, and then applying acoustic pressure and particle velocity continuities at $x_1 = \ell_n$ or $x_3 = 0$, Eqs. (27) and (28) can be combined into an impedance matrix as

$$\{ P_i(x_1 = 0) \} = [T_3] \{ u_i(x_1 = 0) \}. \quad (29)$$

The average of acoustic pressure and particle velocity at nodes yields the acoustic impedance of the Helmholtz resonator as a function of frequency. The resonance frequency at which the acoustic impedance is minimum can be calculated.

B. Pipe-mounted model

For a pipe-mounted model as shown in Fig. 3, the main duct (M) and neck (I) form an acoustic domain (M+1) and the relationship between acoustic pressure and velocity at the boundaries is given by

$$\begin{bmatrix} P_{M+1}^i \\ P_{M+1}^o \\ P_{M+1}^{\ell_n} (x_1 = \ell_n) \end{bmatrix} = [T_3] \begin{bmatrix} u_{M+1}^i \\ u_{M+1}^o \\ u_{M+1}^{\ell_n} (x_1 = \ell_n) \end{bmatrix}, \quad (30)$$

where superscripts $i$ and $o$ denote inlet and outlet of the main duct, respectively, and $r$ the rigid walls. Using rigid boundary condition at the walls of the main duct and neck, $u_r^{i+1} = 0$, and then applying acoustic pressure and velocity continuity at $x_1 = \ell_n$, Eqs. (28) and (30) yield

$$\begin{bmatrix} P^i \\ P^o \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P^i \\ P^o \end{bmatrix}, \quad (31)$$

which defines the transfer matrix elements, $T_{ij}$. Assuming a main duct with constant cross-sectional area, transmission loss can then be calculated from the transfer matrix by

$$TL = 20 \log_{10} \left( \frac{1}{2} |T_{11} + T_{12} + T_{21} + T_{22}| \right). \quad (32)$$

The mesh for BEM calculations, with a typical size of 2.5 cm, are generated using IDEAS. Figure 4 shows a sample mesh for the Helmholtz resonator with $\ell_2 = 10$ cm and 10° expanded conical extension.

IV. RESULTS AND DISCUSSION

A cylindrical Helmholtz resonator has been fabricated with fixed radius $a_2 = 7.62$ cm, length $\ell_c = 20.32$ cm, and a base neck of length $\ell_1$ and fixed radius $a_1 = 2.00$ cm. For the extended neck into the chamber, four configurations with various extension lengths and shapes are considered as summarized in Table I: configuration 1—base neck with fixed length of $\ell_1 = 8.5$ cm and straight extensions with different lengths, $\ell_2 = 0 – 18$ cm; configuration 2—fixed total (base + extended) neck length of $\ell_n = \ell_1 + \ell_2 = 18.5$ cm and varying extension length from $\ell_2 = 0$ to 18 cm, with corresponding base neck length from $\ell_1 = 18.5$ to 0.5 cm; configuration 3—base neck with fixed length of $\ell_1 = 8.5$ cm and neck extension with $\ell_2 = 10$ cm and conical shapes (5° contraction or 10° expansion); and configuration 4—base neck with fixed length of $\ell_1 = 8.5$ cm, and neck extension with $\ell_2 = 10$ cm and perforations. The main duct is built of square cross-section (4.3 cm $\times$ 4.3 cm) for clear identification of the neck.
TABLE I. Helmholtz resonators with various neck lengths [cm] and shapes.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Extended neck shape</th>
<th>Base neck length ($\ell_1$)</th>
<th>Neck extension length ($\ell_2$)</th>
<th>Total neck length ($\ell_n=\ell_1+\ell_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Straight solid</td>
<td>8.5</td>
<td>0–18</td>
<td>8.5–26.5</td>
</tr>
<tr>
<td>2</td>
<td>Straight solid</td>
<td>18.5–0.5</td>
<td>0–18</td>
<td>18.5</td>
</tr>
<tr>
<td>3</td>
<td>Conical straight solid</td>
<td>8.5</td>
<td>10</td>
<td>18.5</td>
</tr>
<tr>
<td>4</td>
<td>Straight perforated</td>
<td>8.5</td>
<td>10</td>
<td>18.5</td>
</tr>
</tbody>
</table>

FIG. 5. Resonance frequency predictions for Helmholtz resonators with various neck extension lengths ($\ell_1=8.5$ cm).

FIG. 6. Measured transmission loss for Helmholtz resonators with neck extension ($\ell_1=8.5$ cm).
length. The square main duct is then connected to the circular impedance tube with smooth transitions that retain a constant cross-sectional area development.

For a piston-driven Helmholtz resonator of configuration 1, Fig. 5 compares a number of predictions for resonance frequencies. In general, the resonance frequency decreases as the extension length \( \ell_2 \) increases from 0 to 18 cm. 2D analytical and BEM predictions exhibit a good agreement, as expected. The difference between the resonance frequencies using 2D analytical approach with five and ten higher order modes is within 0.3%, thus only the predictions with five modes are presented. Setting \( s=0 \) in Eqs. (16)–(20) gives 1D axial model predictions, leading to overestimation of resonance frequencies. Lumped model predictions without end correction demonstrate higher resonance frequencies than the other models. In the lumped model, the volume of a neck extension is subtracted from the total resonator volume. The relatively large difference for short extensions among the analytical methods may be attributed to increasing relative significance of higher order modes (due to decreasing total neck length) generated at the interface between neck and volume.

An impedance tube test setup is used to obtain transmission loss of pipe-mounted resonators applying the two-microphone technique. Figure 6 shows the measured transmission loss for different neck extension lengths. As the extension length increases, the resonance frequency decreases and the attenuation band becomes narrower. Note that a 15 cm change in the extension length results in a 33 Hz

FIG. 7. Comparison of predictions with experiment for a Helmholtz resonator without neck extension (\( \ell_1=8.5 \text{ cm and } \ell_2=0 \text{ cm} \)).

FIG. 8. Comparison of predictions with experiment for a Helmholtz resonator with neck extension (\( \ell_1=8.5 \text{ cm and } \ell_2=15 \text{ cm} \)).
shift in the resonance frequency, which is a significant variation at low frequencies considered here.

In addition to the resonance frequency, transmission loss of the pipe-mounted Helmholtz resonator can be predicted using Eqs. (23) and (32). For a Helmholtz resonator without and with (\(\ell_2 = 15\) cm) neck extension [Figs. 1(a) and (b), configuration 1], Figs. 7 and 8 compare such transmission loss predictions with experiments. Since the transmission loss behavior for \(\ell_2 = 5\) and 10 cm is similar to Figs. 7 and 8 except for the resonance frequencies, the comparisons for \(\ell_2 = 5\) and 10 cm are excluded. While the 2D analytical predictions shift to higher frequencies by 0.5–2 Hz, the 3D BEM shows a close agreement with the experiments. The difference between the 2D analytical method and 3D BEM is due to the neglect of higher order modes in the former at the neck and main duct interface. As the neck extension length \(\ell_2\) increases, the differences tend to diminish since the total neck length increases. Both 2D analytical and BEM have peaks higher than the experiment since damping or dissipation effects are neglected in the predictions.

For a fixed total neck length (configuration 2) \(\ell_n = 18.5\) cm, the predicted resonance frequency shifts as a function of the extension length \(\ell_2\) as shown in Fig. 9, which include lumped, 1D, and 2D analytical models. For a given geometry, the variation in resonance frequency remains within 3 Hz for all analytical approaches.

Figure 10 considers a 10-cm-long conical neck extension [Fig. 1(c), configuration 3] and compares the BEM predictions with experiments. Due to the complexity of the geometry, only the BEM predictions are presented here.

FIG. 9. Predicted resonance frequencies for Helmholtz resonators with fixed total length, \(\ell_n = \ell_1 + \ell_2 = 18.5\) cm.

FIG. 10. Comparison of BEM predictions with experiment for a Helmholtz resonator with conical neck extension (\(\ell_1 = 8.5\) cm and \(\ell_2 = 10\) cm).
Compared to the one with constant cross-sectional area extension, the resonator with 10° expansion has increased the resonance frequency and broadened the attenuation band, whereas the 5° contraction has lowered the resonance frequency and narrowed the attenuation band. The directional behavior may be qualitatively interpreted in terms of increasing and decreasing effective cross-sectional areas, respectively. The BEM predictions capture these trends with satisfactory accuracy.

As demonstrated in Figs. 6–10, the resonance frequency and transmission loss behavior can be modified by changing the neck length or shape. The resonance frequency can be further controlled by the perforation of the neck extension. Figure 11 shows transmission loss predicted by BEM for Helmholtz resonators with perforated neck extension [Fig. 1(d), configuration 4]. The top of the extension is fully open to the cavity volume and the cylindrical extension with $\ell_2 = 10$ cm is now perforated with porosity $\sigma$. As $\sigma$ increases, the resonance frequency increases, and the magnitude of the transmission first decreases followed by an increase. Sullivan’s empirical expression is used for perforation impedance in these predictions. As $\sigma$ increases, the extension length effectively varies from $\ell_2 = 10$ cm to $\ell_2 \approx 0$ cm at high porosities, therefore the corresponding resonance frequency varies from the limit of solid extension to the no-extension case with accompanying increase in frequency. Also, as $\sigma$ varies from 1 to 99%, the resistance or damping of the perforation impedance decreases, leading to an increase in transmission loss.

V. CONCLUDING REMARKS

Acoustic characteristics of a Helmholtz resonator with extended neck into cavity have been studied analytically, numerically, and experimentally. Predictions by lumped, 1D axial, 2D analytical, and 3D BEM models are compared to each other and the experimental results. Extension of neck into the cavity lowers the resonance frequency substantially and narrows the transmission loss band. With the total neck lengths retained, the shape of the neck extension, e.g., conical contraction and expansion, also changes the resonance frequency and transmission loss. Thus, modification of the neck extension length or shape may be an effective method to control the resonance frequency of a Helmholtz resonator without changing the cavity volume. Also adding perforation to the neck extension can change the resonance frequency and transmission loss behavior.

For a piston-driven Helmholtz resonator, the 2D analytical method shows a good agreement with BEM, while simplified lumped and 1D models overpredict the resonance frequencies. For a pipe-mounted resonator, transmission loss predictions by 2D analytical model show reasonable agreement with experiments. The deviation of 2D analytical transmission loss prediction from experiment is mainly due to the neglect of higher order modes in the vicinity of neck and main duct interface. As the total neck length increases, the difference between 2D analytical method and experiments decreases.

The extension of neck into the cavity can shift the resonance frequency down without increasing the volume. Changing the length and shape of or adding perforations to the neck extension can move the resonance frequency and modify the transmission loss behavior. Such trends can be appealing in noise control applications where designers may be limited by the space constraints.

APPENDIX A: INTEGRATION IN Eqs. (18)–(20)

The following integral relations for Bessel functions are considered (Abramowitz and Stegun):

$$\int x J_0(\alpha x) J_0(\beta x) \, dx = \frac{x\{\alpha J_1(\alpha x) J_0(\beta x) - \beta J_0(\alpha x) J_1(\beta x)\}}{\alpha^2 - \beta^2}.$$  \hspace{1cm} (A1)
\[
\int x J_0(\alpha x)dx = \frac{x^2}{2} \{J_0'(\alpha x)\}^2 + \frac{x^2}{2} \{J_0(\alpha x)\}^2 - \frac{x^2}{2} \{J_1(\alpha x)\}^2 + J_0^2(\alpha x),
\]
where \( J_\alpha \) and \( Y_\alpha \) are the first and second kind Bessel functions, respectively. The integration denoted by \( \langle \_ \_ \_ \_ \_ | \_ \_ \_ \_ \_ \rangle \) in Eqs. (18)–(20) can be calculated from Eqs. (A1)–(A3) and the orthogonality of Bessel functions as follows:

\[
\langle \psi_{\lambda;\sigma}, \psi_{\lambda;\sigma} \rangle = \int_0^a r J_0^2 \left( \frac{\alpha r}{a} \right) dr = \frac{a^2}{2} J_0^2(\alpha), \quad (A4)
\]

\[
\langle \psi_{\lambda;\sigma}, \psi_{\lambda;\lambda} \rangle = \begin{cases} 
\int_0^a r J_0(\alpha r/a_1) J_0(\alpha r/a_2) \frac{\alpha_s J_0(\alpha_s) J_1(\alpha_s a_1) J_0(\alpha_s a_2)}{\left( \frac{\alpha_s}{a_1} \right)^2 - \left( \frac{\alpha_s}{a_2} \right)^2} \left( \alpha_s \neq \alpha_\lambda \right) & (A5) \\
\int_0^a r J_0^2(\alpha r/a_1) \frac{\alpha_s}{a_1} \left( \frac{\alpha_s}{a_2} = \frac{\alpha_s}{a_1} \right), & (A6)
\end{cases}
\]

\[
\langle \psi_{\lambda;\sigma}, \psi_{\lambda;\lambda} \rangle = \begin{cases} 
\int_0^a r J_1(\beta r/a_1) Y_0(\beta r/a_2) \frac{J_0(\beta r/a_1) J_0(\beta r/a_2)}{\left( \frac{\beta}{a_1} \right)^2 - \left( \frac{\beta}{a_2} \right)^2} \left( \alpha_s \neq \beta_s \right) & (A7) \\
\int_0^a r J_0^2(\alpha r/a_1) \frac{\alpha_s}{a_1} \left( \frac{\alpha_s}{a_2} = \frac{\alpha_s}{a_1} \right), & (A8)
\end{cases}
\]

\[
\langle \psi_{\lambda;\sigma}, \psi_{\lambda;\lambda} \rangle = \begin{cases} 
\int_0^a r J_0(\alpha r/a_1) J_0(\alpha r/a_2) \frac{\alpha_s J_0(\alpha_s) J_1(\alpha_s a_1) J_0(\alpha_s a_2)}{\left( \frac{\alpha_s}{a_1} \right)^2 - \left( \frac{\alpha_s}{a_2} \right)^2} \left( \alpha_s \neq \alpha_\lambda \right) & (A9) \\
\int_0^a r J_0^2(\alpha r/a_1) \frac{\alpha_s}{a_1} \left( \frac{\alpha_s}{a_2} = \frac{\alpha_s}{a_1} \right), & (A10)
\end{cases}
\]
16 M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions (Dover, New York, 1970).