ABSTRACT

The behavior of the compression system in turbochargers is studied with a one-dimensional engine simulation code. The system consists of an upstream compressor duct open to ambient, a centrifugal compressor, a downstream compressor duct, a plenum, and a throttle valve exhausting to ambient. The compression system is designed such that surge is the low mass flow rate instability mode, as opposed to stall. The compressor performance is represented through an extrapolated steady-state map. Instead of incorporating a turbine into the model, a drive torque is applied to the turbocharger shaft for simplification. Unsteady compression system mild surge physics is then examined computationally by reducing the throttle valve diameter from a stable operating point. Such an increasing resistance decreases the mass flow rate through the compression system and promotes surge. Mild surge is predicted as the mass flow rate is decreased below the stability limit, with oscillations of mass flow rate and pressure exhibited at the Helmholtz resonance frequency of the compression system. The computational results are shown to be able to reproduce the experimental observations available in the literature.

1. INTRODUCTION

Away from (to the right of) the surge line, the pressure ratio of the compressor increases in a stable mode as the mass flow rate is reduced. As the operating condition approaches the surge line, the pressure rise reaches its maximum value and a further reduction in mass flow causes a sudden change in the flow pattern of the compressor [1], leading to either stall or surge. The stall in centrifugal compressors manifests itself in different modes, including (1) full- and part-span rotating stall, (2) axisymmetric stall near the inducer tips, and (3) stationary non-axisymmetric stall produced by downstream asymmetry of the volute [1]. These types of stall result in a redistribution of the flow field and a lower pressure ratio as the annulus average mass flow rate is reduced beyond the peak pressure rise. Stalling is often tolerated in centrifugal compressors without a drastic decrease in performance since the majority of pressure rise is attributed to the centrifugal effects which occur in the presence of stall cells and separated flow.

Surge, on the other hand, is the low-flow behavior of greatest concern when operating a centrifugal compressor near the surge line. It is an axisymmetric, self-excited system oscillation which can be categorized as “mild” or “deep” depending on the degree of mass flow fluctuation. “Mild surge” represents the conditions where the annulus average mass flow oscillates but remains in the forward direction at all times. Such oscillations are characterized by the Helmholtz frequency

\[ f_H = \frac{a}{2\pi V_p L_c} \sqrt{\frac{A_c}{V_p L_c}} \]  

(1)

of the compression system [2], where \( a \) is the speed of sound, \( V_p \) is the volume of the compression system plenum, \( A_c \) is the equivalent cross-sectional compressor duct area (inducer eye area), and \( L_c \) is the equivalent length of the duct. One manifestation of surge is the change in noise. If the mass flow oscillations are severe and the mean flow reverses its direction during part of the cycle, the compressor has entered “deep surge” which can be detrimental to both the turbocharger and engine. The dominant frequency is dictated
by the plenum emptying and filling times and is typically well below the Helmholtz resonance of the compression system.

Zero-dimensional (lumped parameter) models have been developed to predict the surge behavior of both axial [3] and centrifugal [4] compression systems with reasonable accuracy. These models formulated a set of nonlinear equations to estimate the system dynamics in an upstream compressor duct open to ambient, a compressor, a downstream compressor duct, a plenum, and a throttle valve exhausting to ambient. Greitzer’s approach [3] has been used extensively in literature to estimate the operating point for surge inception and the subsequent oscillations. This analysis has revealed a dimensionless number defined by

$$B = \frac{U}{2a} \sqrt{\frac{V_p}{A_i L_c}},$$

where $U$ is the impeller blade tip speed, hence $B$ is a function of the impeller blade tip Mach number as well as the geometry of the compression system. Above a critical value of $B$, surge becomes the instability mode encountered, as opposed to stall. The measured stall/surge boundary in Greitzer’s work was around $B$=0.8. Hansen et al. [5] demonstrated that Greitzer’s model could be extended to centrifugal compressors. The approach was further advanced by Fink et al. [6] to include the shaft speed dynamics for a surging centrifugal compressor (a high-speed radial impeller with vaneless diffuser surrounded by a volute) as an integral part of the cycle. A method for implementing the foregoing lumped model into a zero-dimensional engine simulation code has been outlined by Theotokatos and Kyratsos [7]. Galindo et al. [8] incorporated a surge model into their one-dimensional (1-D) in house gas-dynamic code. They were able to predict the amplitude and dominant frequency of fluctuations in the compressor exit pressure during deep surge with reasonable accuracy. However, their mild surge predictions underestimated the dominant frequency and overestimated the amplitude of pressure oscillations.

The objective of this study is to reproduce the unsteady physics of mild surge on a turbocharger bench setup with a 1-D engine simulation code. This investigation focuses on predicting the compressor operating point for instability (mild surge) inception along with the amplitude and frequency of the resulting oscillations. The large $B$ compression system ducting from the turbocharger test bench of Fink et al. [2] is modeled using a commercially available engine simulation code GT-Power [9]. A schematic of the compression system geometry is shown in Fig. 1. This system is designed with a large plenum such that $B$=2.7 at the speed of interest (48k rpm) and surge is the low-flow instability mode encountered, as opposed to stall. The compressor map data from both the small and large $B$ systems of Fink et al. is combined, extrapolated and interpolated using a preprocessor developed in the present study and input as a text file to the code. The oscillations of the nondimensionalized mass flow rate, pressure ratio, and impeller tip speed from the current study are compared with experimental observations of Fink et al. to evaluate the accuracy of predictions. A detailed analysis is then presented for pressure, mass flow rate, and temperature at key locations, providing further insight into the physics throughout the compression system.

Following this introduction, Section 2 illustrates the source of the “Helmholtz resonator” frequency. Section 3 describes the compressor map and Section 4 the 1-D compression system model. Simulation results are then compared with the experimental observations of Fink et al. [2] in Section 5. Section 6 demonstrates the influence of the compression system geometry on stability, followed by concluding remarks in Section 7.
2. INHERENT HELMHOLTZ RESONANCE

A compression system on a turbocharger test bench typically consists of a compressor operating between two circular ducts. The duct upstream of the compressor is open to ambient and the downstream duct exits into a plenum (such as an intercooler for centrifugal and burner for axial compressors) of larger cross-sectional flow area. The flow exiting the plenum then passes through a restriction (such as a throttle or intake valve for centrifugal and turbine for axial compressors) with a flow area considerably smaller than the plenum. This configuration has inspired a lumped parameter approach that models the surge oscillations in a manner analogous to those of a Helmholtz resonator [2,3,10]. The frequency of such oscillations is unique to the duct-plenum (cavity) coupling in terms of inertia in the duct balanced by pressure forces due to compressibility in the cavity. As expected, oscillations of pressure and mass flow in the compression system occur at the inherent Helmholtz resonator frequency (even) in the absence of the compressor. This unsteady behavior is demonstrated using a 1-D engine simulation code by replacing the compressor in Fig. 1 with a duct of equivalent length. A flow is generated by applying a pressure gradient across the boundaries. After the flow stabilizes, the throttle valve is fully closed to provide a perturbation leading to oscillations of mass flow and pressure, as shown in Fig. 2. A frequency domain analysis of pressure oscillations shows the dominant frequency to be 7.0 Hz, which is close to the theoretical Helmholtz resonator frequency of 7.3 Hz calculated using Eq. (1).

3. COMPRESSOR

A representation of the compressor, by user provided performance maps, needs to be integrated into the ducting of the air flow system prior to surge predictions. This is accomplished in GT-Power (hereafter referred to as the “code”) by providing performance data in the form of mass flow rate, pressure ratio, and efficiency at constant rotational speeds. This information is either entered into the ‘CompressorMap’ object or referenced as an external text file (“.cmp” file). Both methods use rotational speed and pressure ratio as the inputs to the lookup map, and mass flow rate and efficiency are the outputs. When data is input to the ‘CompressorMap’ object (the first approach), the preprocessor of the code extrapolates the data to choke, zero speed, higher speeds (optional), and reverse flow (optional). Then, the map is interpolated between the user provided data points. The current study employs the alternative method (the second approach) of providing compressor data as an external text file with a ‘.cmp’ extension, which requires the user to perform all interpolations and extrapolations, and, as a result, the preprocessor of the code is not used. A MATLAB script was also developed in the present study to perform all of the preprocessing and to formulate the compressor information into a text file with the “.cmp” format required by the code. For further information regarding the “.cmp” compressor map format the User's Manual may be referred [11].

The compressor map used in the present study was re-created from Fink's [2,6] small and large B compressor characteristics and Fink’s forward flow nondimensional torque $\Gamma_c$ as a function of flow coefficient $\phi_c$ as shown in Figs. 3 and 4, respectively. The published compressor map contains data at six constant rotational speeds in the range of 25k to 51k rpm with the large B system data in blue and the small B data in
red, as shown in Fig. 3. The “Large B Surge Line” (hereafter simply referred to as the “surge line”) designates the deep surge boundary for the large B compression system shown in Fig. 1. To the right of the surge line, the small and large B data are identical, illustrating that the compressor characteristics are independent of the compression system ducting (a property of the compressor alone), yet the coupled compression system (compressor, ducting, and throttle) dictates the stable portion of the map available for use. The compressor nondimensional torque $\Gamma_c$ is calculated from map data as

$$\Gamma_c = \frac{\tau_c}{\rho_0 A c_p U^2} = \frac{\frac{m_c c_p T_{1t}}{\rho_0} \left( \frac{\gamma - 1}{PR_c^{\gamma - 1}} - 1 \right)}{\rho_0 A c_p r_2^2} ,$$

(3)

where $\tau_c$ is the torque driving the compressor (Baines [12]), $\rho_0$ is the density of air at ambient conditions, $r_2$ is the radius of the impeller tip, $c_p$ is specific heat of air at constant pressure, $m_c$ is the mass flow rate through the compressor, $T_{1t}$ is the total inlet temperature, $\omega$ is the angular velocity of the impeller, $PR_c$ is the total-to-total pressure ratio of the compressor, and $\gamma$ is the ratio of specific heats. The compressor flow coefficient $\phi_c$ represents the mass flow rate non-dimensionalized as

$$\phi_c = \frac{m_c}{\rho U} ,$$

(4)

where $\rho$ is the density at the compressor inlet (for forward flow). The $\Gamma_c$ vs. $\phi_c$ data shown in Fig. 4 provides a convenient means for extrapolating compressor isentropic efficiency $\eta_c$ to zero mass flow rate since the data at all rotational speeds nearly collapses onto a single curve.

The compressor characteristics of Fink are extrapolated here to cover the entire forward flow operating region of the compressor including down to zero speed, up to 60k rpm, to zero flow, and to choke. The pressure ratio of each constant speed line at zero mass flow rate $PR_0$ is calculated using radial equilibrium theory and assuming an isentropic process [7]

$$PR_0 = C_{PR} \left[ 1 + \frac{\gamma - 1}{2 \gamma R T_{1t}} \omega^2 \left( r_2^2 - r_1^2 \right) \right]^{\gamma - 1} ,$$

(5)

where $C_{PR}$ is a constant multiplier, $R$ is the gas constant for air, and $T_{1t}$ is the compressor inlet temperature. $r_1$ is the mean geometric radius of the impeller eye, which divides the impeller eye area into two sections with the same area and is calculated as

$$r_1 = \sqrt{\frac{r_{1t}^2 + r_{1h}^2}{2}} ,$$

(6)

where $r_{1t}$ and $r_{1h}$ are the radii of the inducer tip and hub, respectively. For the present study, a $C_{PR}$ value slightly larger than unity was applied to $PR_0$ in Eq. (5) to match the
experimental data. This multiplier was 1.04 for the lowest speed (25k rpm) and 1.09 for the highest compressor speed (51k rpm).

Since the rare small $B$ system data is available for Fink's compressor, the present study uses higher order polynomials to fit the small $B$ constant speed lines down to zero mass flow rate. This method allows the constant speed lines used in the model to obtain a nearly perfect fit to the experimental data. If the rare small $B$ system data were not available, the Moore and Greitzer [13] method would have been used to extrapolate the compressor characteristics from the peak pressure rise to zero mass flow rate, as described in Appendix A.

The compressor map data must also be extrapolated down to zero speed, to choke, and optionally to higher speeds. Extrapolation to both lower and higher speeds is performed on the nondimensional characteristics. The compressor flow coefficient $\phi_c$ is calculated using Eq. (4), and the nondimensional compressor isentropic head coefficient involves the nondimensional pressure as

\[
\psi_c = \frac{\Pi_c^{(\gamma - 1)/\gamma} - 1}{(\gamma - 1) M_{a_{t,0}}^2},
\]

where

\[
\Pi_c = \frac{p_{3t}}{p_0},
\]

$p_{3t}$ being the total pressure at the compressor exit and $p_0$ the pressure at ambient conditions, and

\[
M_{a_{t,0}} = \frac{U}{a_0}
\]

is the compressor exit tip Mach number, with $a_0$ being the speed of sound at ambient conditions. Note that the nondimensional compressor characteristics ($\psi_c$ vs. $\phi_c$) nearly collapse onto a single curve, partially removing the speed dependence, as shown in Fig. 5. The extrapolation of map data to lower speeds is achieved by performing a speed weighted linear interpolation between the nondimensional form of the lowest constant speed line (25k rpm) and zero speed, assuming zero mass flow rate, pressure rise, and efficiency when the impeller is not rotating. Extrapolation of the compressor data to higher speeds is completed by transforming the nondimensional data of the highest constant speed line (51k rpm) into dimensional form at higher speeds. The compressor data usually requires interpolation at intermediate speeds and details regarding the approach used in the current study are included in Appendix B.

The final version of the compressor map used for the code is shown in Fig. 6. The large and small $B$ compression system data of Fink is superimposed on the map along with the corresponding constant speed lines to illustrate the comparison. The MATLAB script writes the compressor map information to a text file with the required “.cmp” format.

A method similar to Theotokatos and Kyrtatos [7] is used to extrapolate the compressor efficiency down to zero mass flow rate. The nondimensional torque $\Gamma_c$ vs. flow coefficient $\phi_c$ data in Fig. 4 is extrapolated by applying a quadratic curve fit in the form

\[
\Gamma_c = C_0 + C_1 \phi_c + C_2 \phi_c^2,
\]

with $C_0$, $C_1$, $C_2$ being the fit constants. The extrapolated compressor efficiency is calculated by substituting Eq. (10) into Eq. (3) and using discrete values of $\dot{m}_c$ and $PR_c$ from the extrapolated compressor characteristics. The compressor map usually requires interpolation at intermediate speeds and details regarding the approach used in the current study are included in Appendix B.

The final version of the compressor map used for the code is shown in Fig. 6. The large and small $B$ compression system data of Fink is superimposed on the map along with the corresponding constant speed lines to illustrate the comparison. The MATLAB script writes the compressor map information to a text file with the required “.cmp” format.

Since limited data is available for this compressor, the nondimensional torque (used to calculate efficiency) was assumed to be independent of speed. As a result, the compressor efficiency is only a factor of the flow coefficient at speeds above 25k rpm. This results in constant peak efficiency along each constant speed line. If more data were to be available, Eq. (10) could be used to fit the $\Gamma_c$ vs. $\phi_c$ data at each constant speed individually and incorporate a speed-dependent efficiency. 
4. ONE-DIMENSIONAL COMPRESSION SYSTEM MODEL

The numerics describing unsteady, compressible fluid flow have been incorporated into 1-D engine simulation codes since the late 1970's. The commercial code used here for the current study solves the non-linear balance equations of mass, momentum and energy, along with the equation of state, using an explicit time integration method. These equations are applied to spatially discrete control volumes within the air flow system. The 1-D mass conservation is represented by

\[ \frac{dm}{dt} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}, \]  

(11)

where \( m \) is the mass of the discrete volume, and \( \dot{m} \) is the mass flow rate across the control volume boundaries. The 1-D momentum conservation may be expressed as

\[ \frac{d\dot{m}}{dt} dx = A \dot{p} + \sum_{\text{in}} \left( \dot{m}u \right) \text{out} - \sum_{\text{out}} \left( \dot{m}u \right) \text{in} - 4f \frac{\rho u |u|}{2} \frac{A dx}{D} - K \left( \frac{1}{2} \rho u |u| \right) A, \]  

(12)

where \( \rho \) is the pressure, \( A \) is the cross-sectional area of flow, \( u \) is the velocity at the boundary of the control volume, \( f \) the friction coefficient, \( \rho \) is the density, \( D \) is the equivalent diameter, and \( K \) is the pressure loss coefficient. The 1-D energy conservation is expressed as

\[ \frac{d(e \rho \dot{m})}{dt} = p \frac{dV}{dt} + \sum_{\text{in}} (e \dot{m}h) - \sum_{\text{out}} (e \dot{m}h) - h_c A_s (T_f - T_w), \]  

(13)

where \( e \) is the total internal energy (internal energy plus kinetic energy) per unit mass, \( V \) is the volume, \( h \) is the specific enthalpy, \( h_c \) is the heat transfer coefficient, \( A_s \) is the heat transfer area, \( T_f \) is the fluid temperature, and \( T_w \) is the wall temperature. The solutions of Eqs. (11), (12), (13) in combination with the equation of state yield the scalar fluid properties within each control volume (pressure, temperature, density, internal energy, enthalpy, etc.) and the vector properties at the boundaries (mass flow rate, velocity, etc.).

The compression system of Fink is modeled in the code with the compressor inlet pipe and throttle exit connected to ambient boundary conditions. The compressor performance data shown in Fig. 6 is input to the ‘Compressor’ object as a text file in “.cmp” format. Instead of incorporating a turbine into the model, a drive torque is applied to the turbocharger shaft for simplification. The turbocharger rotational inertia is often artificially manipulated by use of a multiplier to speed up convergence of steady-state simulations. It is important that the inertia multiplier is maintained equal to one throughout simulations near the surge line, as unsteady surge oscillations may be encountered. This is especially important for turbocharged engine models where pressure pulsations in the exhaust and induction systems cause speed fluctuations.

Among the options GT-Power offers for a time constant to dampen changes in compressor mass flow rate, the present study has chosen

\[ \tau_{GTP} = \frac{n}{\omega}. \]  

(14)

This time constant is proportional to the time required for a number of rotor revolutions \( n \), and a value of \( n=2 \) is used in the current study. If the damping is not present, a small change in pressure ratio at the peak of the characteristic can cause an unrealistically large change in mass flow rate in a single time-step. This damping also allows the compressor to deviate from the steady-state map during unsteady surge cycles.

Incorporating surge prediction capabilities into a 1-D engine simulation code offers the following benefits compared to the lumped parameter models of Greitzer [3] and Fink et al. [2] that have been used extensively in literature. First, the nonlinear formulation with spatial distribution allows the wave dynamics of the compression system to be modeled and is therefore advantageous for engine simulations. Second, many of the major simplifying assumptions required for the lumped parameter (0-D) model of Fink et al. can be eliminated, including: (a) incompressible flow in the compressor duct, (b) isentropic plenum expansion or compression, (c) choked throttle valve, (d) short throttle duct length so that the inertia can be ignored, and (e) negligible
velocity in the plenum. A couple of the major simplifying assumptions of the lumped parameter model that are retained within the 1-D approach include: (a) discontinuity of pressure and density across the compressor, which is modeled as an actuator disk, and (b) negligible gas angular momentum in the compressor passages compared to the impeller angular momentum.

5. MILD SURGE SIMULATION RESULTS

The simulations in the current study are aimed at reproducing the experimental mild surge observations of Fink's [2,6] large B compression system at a time average rotational speed of 48k rpm \( (Ma_{t,0} = 0.92) \). His data is shown in Fig. 7a with repeating oscillations of the nondimensional parameters \( \phi_c \), \( Ma_{t,0} \), and plenum isentropic head coefficient \( \psi_p \) where \( \Pi_p \) is the ratio of total plenum pressure to ambient pressure. The \( \phi_c \) data fluctuates with an amplitude of 20% of the mean flow \( (0.046) \) about a time mean value of 0.23, and the dominant frequency of oscillations is reported as 7.3 Hz. The compressor operating conditions on the surge line at 48k rpm are \( \dot{m}_c = 0.30 \text{ kg/s} \) \( (\phi_c = 0.225) \) and \( PR_c = 1.98 \) \( (\psi_p = 0.637) \), and this marks the deep surge boundary of Fink's experimental results. For the simulations performed here, the compressor is brought to a stable operating condition near the stability limit at 48k rpm. Then, the throttle at the compression system exit is partially closed to move the compressor operating point into the surge region. When the stability limit of the compression system is crossed, low amplitude mild surge oscillations appear and grow to the converged results presented in Fig. 7b for comparison with the experimental results of Fink. For ease of such comparisons, Fig. 7b has retained both the horizontal and vertical axis scales the same as in Fig. 7a. The simulation is stopped when the cycle-to-cycle change in mass flow rate amplitude is extremely small (less than \( 1 \times 10^{-5} \text{ kg/s} \) here) to ensure that the compression system does not enter deep surge. The oscillations of \( \phi_c \) are predicted to occur at a mean value of 0.22 with an amplitude of 0.046 (21% of the mean flow) and appear to be nearly identical to the results of Fink. Similarly, the \( Ma_{t,0} \) and \( \psi_p \) predictions almost exactly reproduce the amplitude, frequency, and the time averaged operating point of the corresponding experimental results. These comparisons demonstrate the ability of nonlinear 1-D time-domain approaches to predict the compressor operating conditions at the instability inception. Frequency-domain analysis of the computed \( \phi_c \) and \( \psi_p \) oscillations is shown in

\[
\psi_p = \frac{\Pi_p^{(\gamma-1)/\gamma} - 1}{(\gamma-1)Ma_{t,0}^2},
\]

\[\text{(15)}\]

Figure 7. Mild surge results: (a) Experimental data of Fink and (b) Simulation from the present study.
Fig. 8. The dominant fundamental frequency is predicted to be 7.3 Hz, and the somewhat steepened wave forms lead to additional frequency content at harmonics of the fundamental frequency. The predicted fundamental frequency is identical to the measured result reported by Fink. The theoretical Helmholtz resonance frequency may be calculated from Eq. (1) as

\[
f_{H} = \frac{a}{2\pi} \sqrt{\frac{A_c}{V_p L_c}} = \frac{394 \text{ m/s}}{2\pi} \times \sqrt{\frac{3.58 \times 10^{-3} \text{ m}^2}{(0.208 \text{ m}^3)(1.267 \text{ m})}} = 7.3 \text{ Hz},
\]

where the speed of sound is based on the mean plenum temperature of 387 K. The equivalent duct length

\[
L_c = \left[ \int_0^l \frac{dl}{A(l)} \right] A_c = \left( \frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right) A_c = (354 \text{ m}^{-1})(3.58 \times 10^{-3} \text{ m}^2) = 1.267 \text{ m}
\]

is calculated using the compression system geometry in Fig. 1 and taking the equivalent duct area (reference area) as the cross-sectional area of the inducer eye. The additional frequencies have also been observed experimentally by Gravdahl et al. [14].

Figure 10 shows the dimensionless compressor characteristic used in the present model with the data of Fink. Note that the flow coefficient \( \phi_c \) on the surge line at 48k rpm is equal to 0.225. Mild surge operating points in the compressor (\( \psi_c \) vs. \( \phi_c \) in green symbols) and plenum (\( \psi_p \) vs. \( \phi_c \) in blue symbols) oscillate in a counter-clockwise direction on the nondimensional compressor map. The change of compressor and plenum fluctuation shapes from dimensional (Fig. 9) to nondimensional (Fig. 10) maps is partially due to the different definitions of nondimensional pressure and pressure ratio. \( \Pi \) is relative to a constant reference pressure and is used for calculating \( \psi_c \) and \( \psi_p \) in Eqs. (7) and (15), respectively, while \( PR \) is relative to the fluctuating compressor inlet pressure. The other factor contributing to the shape difference is the fluctuating rotational speed, which appears as \( Ma_{t,0} \) in the denominator of Eqs. (4), (7), and (15). \( Ma_{t,0} \) is larger when the compressor mass flow rate is accelerating (from Location 3 to 1 in Fig. 9) than when the mass flow rate is decelerating (from Location 1 to 3 in Fig. 9). This contributes to a lower \( \psi_c \) during mass flow acceleration. The compressor plenum (\( PR_p \)) are defined relative to the total pressure at the compressor inlet. Mild surge operating points for the compressor (\( PR_c \) vs. \( \dot{m}_c \) in green symbols) oscillate in the clockwise direction on the map and the plenum (\( PR_p \) vs. \( \dot{m}_c \) in blue symbols) operates along a curved path. At Location 1 of the compressor surge cycle, the operating point is nearly on the peak of the 48k rpm characteristic. As \( \dot{m}_c \) and \( PR_c \) decrease, the operating points follow the constant speed line and the surge line is crossed at Location 2. The mass flow rate and pressure ratio reach a minimum at Location 3, and the compressor speed starts increasing. The compressor flow then accelerates, back across the surge line, until the peak pressure ratio of the cycle is reached at Location 4. Then, the speed and pressure ratio decrease to Location 1 and the cycle is repeated. The compressor and plenum operating points have uniformly spaced time intervals of 0.5 ms, making, for example, the rate of change of \( \dot{m}_c \) and \( PR_c \) clearly visible for the compressor. Note that the discrete operating points are spaced further apart where the compressor mass flow is accelerating, indicating that the compressor moves through this portion of the cycle at a rate faster than when the mass flow rate is decelerating. During mild surge, the compressor spends a significant amount of time to the left of the surge line. In order to predict mild surge, compressor performance data is required in this region of the map where steady-state measurements are not obtainable with the large \( B \) system. This data must either be collected using a system with a smaller \( B \) number (as accomplished by Fink) or estimated through extrapolation. The pressure ratio of the plenum (blue symbols) is lower than that of the compressor because of the pressure losses in the intermediate ducting. The shape of the plenum surge cycles is essentially driven by the pressure wave at the Helmholtz resonator frequency.
operating points nearly follow the nondimensional characteristic while the flow is decelerating, but they operate below the steady-state map when accelerating. The nondimensional plenum operating points follow an elliptical path that appears to be nearly symmetrical about the surge line.

Figure 9. Mild surge compressor and plenum operating points for a single cycle. Selected constant speed lines used in the present study and Fink's surge line are shown for reference.

Figure 10. Nondimensional mild surge compressor and plenum operating points for a single cycle. The nondimensional 48k rpm speed line data of Fink et al. and the nondimensional compressor characteristic at 48k rpm (used in the model) are shown for reference.
To gain further insight into the physics of mild surge, the predicted static pressure, temperature, and mass flow rate are given at four locations within the compression system in Figs. 11, 12, 13, respectively. Locations 1-3 and 7 are specified in Fig. 1, corresponding to the mid-length of the compressor inlet duct, compressor inlet, compressor exit, and mid-length of the plenum, respectively. Figure 11 illustrates that the pressure fluctuations are most severe at the compressor inlet with an amplitude of approximately 2.2 kPa and least severe at the mid-length of the compressor inlet duct (among the locations studied) with an amplitude of 0.25 kPa. A frequency domain analysis of the pressure traces reveals that the maximum sound pressure level (SPL) of about 160 dB occurs at the compressor inlet, corresponding to the dominant mild surge frequency (7.3 Hz), as shown in Fig. 14. The harmonics of the Helmholtz resonances appear throughout the compression system due to non-linearity. It is expected that those harmonics above 20 Hz would become audible. The temperature fluctuations are largest at the compressor exit during mild surge, as shown in Fig. 12, but the amplitude is relatively small at approximately 2.5 K. Mass flow rate fluctuations are severe throughout the compression system, as shown in Fig. 13. The large amplitude mass flow rate fluctuations at Locations 1-3 are nearly identical, and the amplitude in the plenum is lower due to smaller oscillations in velocity as a result of the much larger cross-sectional area of the plenum. The amplitude of mass flow rate oscillations at the compressor is approximately 21% of the mean flow rate. Turbocharger rotational speed is shown to fluctuate with an amplitude of 175 rpm, and this is only 0.4% of the mean value, as shown in Fig. 15. The compressor efficiency fluctuates between the peak value of 73% and about 67% in the present study, as shown in Fig. 16.
6. INFLUENCE OF COMPRESSION SYSTEM GEOMETRY

The compression system in Fig. 1 has a plenum length of 2.38 m and a volume of 0.208 m$^3$, resulting in $B=2.7$ (at 48k rpm) and an instability inception point near the peak pressure rise of the compressor. As $B$ decreases, the compression system can operate free of surge at increasingly positive values of compressor characteristic slope (lower mass flow rates). For illustrative purposes, the volume of the plenum in Fig. 1 is gradually reduced here to determine a critical $B$ value below which the compression system does not enter surge with the engine simulation code. The operating conditions here are identical to those of Section 5, including compressor drive torque, valve diameter, boundary conditions, and the geometry other than the plenum length. A set of runs was performed by sweeping the value of $B$. The predictions reveal that reducing the plenum volume to 4.8% (or, below) of that in Fig. 1 by essentially decreasing the plenum length to 0.115 m, hence $B$ to 0.70 (or, below) eliminates the surge. The stable operating point of the $B=0.70$ compression system is shown by the green dot in Fig. 17. Comparison of Figs. 9 and 17 demonstrates that a compressor operating condition that has originally resulted in mild surge oscillations can be stabilized by adequately reducing the volume of the plenum (therefore $B$).

7. CONCLUSIONS

- The present study has demonstrated the ability to successfully predict compression system mild surge physics with an unsteady nonlinear 1-D solver used in engine simulations.
- The computational results obtained here for mild surge almost exactly reproduce the amplitudes, frequency, and time averaged operating points of the experimental observations.
- For accurate mild surge modeling, it is desirable to employ the experimental data for the entire forward flow region of the compressor (from zero mass flow rate to choke). When experimental data is not available, the compressor performance must be estimated based on extrapolation techniques. The present study implemented a compressor map which was created from experimental data that was extrapolated and interpolated to cover the entire forward flow operating region.
- The Theotokatos and Kyrtatos approach for estimation of the pressure ratio at zero mass flow rate is shown to provide a satisfactory agreement with experimental data, while differing slightly from the measurements at higher rotational speeds for the compressor studied.
A detailed computational analysis of the pressure, temperature, and mass flow rate fluctuations is presented at key compression system locations during mild surge.

The instability inception point of the compressor is also computationally determined, designating the stable operating limit.

The approach described here may be incorporated into turbocharged engine models to assist with the design.

REFERENCES


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DEFINITIONS/ABBREVIATIONS

\( a \) Speed of sound

\( A \) Area of cross-section

\( A_s \) Surface area for heat transfer

\( B \) Nondimensional Greitzer number

\( C_{PR} \) Constant multiplier

\( C_0 - C_2 \) Curve fit constants

\( c_p \) Specific heat of air at constant pressure

\( D \) Equivalent diameter

\( dx \) Length of spatial discretization

\( e \) Total internal energy per unit mass

\( \eta \) Isentropic efficiency

\( f \) Skin friction coefficient
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_H$</td>
<td>Helmholtz resonator frequency</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Ratio of specific heats</td>
</tr>
<tr>
<td>$\Gamma_c$</td>
<td>Nondimensional torque</td>
</tr>
<tr>
<td>$h$</td>
<td>Specific enthalpy</td>
</tr>
<tr>
<td>$h_c$</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>$K$</td>
<td>Pressure loss coefficient</td>
</tr>
<tr>
<td>$l$</td>
<td>Duct length</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Length of equivalent compressor duct</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Mass flow rate</td>
</tr>
<tr>
<td>$Ma_{t,0}$</td>
<td>Mach number of impeller tip</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of rotor revolutions</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular velocity of the shaft</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Flow coefficient</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Nondimensional pressure</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Isentropic head coefficient</td>
</tr>
<tr>
<td>$PR$</td>
<td>Pressure ratio (total-to-total)</td>
</tr>
<tr>
<td>$PR_0$</td>
<td>Pressure ratio at zero mass flow rate</td>
</tr>
<tr>
<td>$R$</td>
<td>Gas constant for air</td>
</tr>
<tr>
<td>$r_1$</td>
<td>Mean geometric radius of the impeller eye</td>
</tr>
<tr>
<td>$r_{1,t}$</td>
<td>Radius of the inducer tip</td>
</tr>
<tr>
<td>$r_{1,h}$</td>
<td>Radius of the inducer hub</td>
</tr>
<tr>
<td>$r_2$</td>
<td>Radius of the impeller tip</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>Torque absorbed by the compressor</td>
</tr>
<tr>
<td>$\tau_{GTP}$</td>
<td>Time constant</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity</td>
</tr>
</tbody>
</table>
\( U \)  
Velocity of the impeller tip

\( V \)  
Volume

**SUBSCRIPTS**

\( 0 - 7 \)  
Location

\( c \)  
Compressor, convection

\( p \)  
Plenum

\( \text{peak} \)  
Peak pressure ratio

\( f \)  
Fluid

\( t \)  
Total property, tip

\( w \)  
Wall
APPENDIX A

MOORE AND GREITZER EXTRAPOLATION METHOD

If the small $B$ compression system map data were not available, the Moore and Greitzer \cite{13} constant speed line extrapolation method would have been applied to the large $B$ data from the peak pressure rise to zero mass flow rate. This method is a cubic polynomial with $PR_c$ as a function of $\dot{m}_c$ and is expressed as

$$PR_c = PR_0 + \left( \frac{PR_{\text{peak}} - PR_0}{2} \right) \times \left[ 1 + \frac{3}{2} \left( \frac{2\dot{m}_c}{\dot{m}_{\text{peak}} - 1} \right) - \frac{1}{2} \left( \frac{2\dot{m}_c}{\dot{m}_{\text{peak}} - 1} \right)^3 \right],$$

**(A1)**

where $PR_0$ is calculated using Eq. (8), $PR_{\text{peak}}$ is the peak pressure ratio on the constant speed line, and $\dot{m}_{\text{peak}}$ is the mass flow rate at $PR_{\text{peak}}$. Equation (A1) forces the extrapolation to pass through $PR_0$ at zero mass flow rate and $PR_{\text{peak}}$ at $\dot{m}_{\text{peak}}$ with zero slope at both locations. Combining Eqs. (8) and (A1), the large $B$ compressor data is extrapolated down to zero mass flow rate to evaluate the Moore and Greitzer method, as shown in Fig. A1. Equation (A1) somewhat underestimates $PR_c$ between the endpoints for the compressor studied. Note again that the Moore and Greitzer method was not used for the current study because custom polynomial fits to the small $B$ data provided a better match.

![Figure A1. Comparison of the Moore and Greitzer extrapolation method with Fink's small B experimental data.](image1)

APPENDIX B

INTERPOLATION FOR INTERMEDIATE ROTATIONAL SPEEDS

The compressor data was interpolated at intermediate map speeds using the following methodology:

1. Each of the extrapolated constant speed line fits are evaluated with an equal number of points. These points are distributed at the same normalized pressure ratio values for each constant speed line, as shown in Fig. B1.

2. The points from two adjacent constant speed lines are nondimensionalized, as shown in Fig. B2.

3. A speed weighted linear interpolation is performed between corresponding points at the same normalized pressure ratio.

4. The interpolated points are dimensionalized as mass flow rate and pressure ratio.

The compressor efficiency is interpolated using a similar procedure.

![Figure B1. Extrapolated constant speed lines with points at equal values of normalized $PR_c$.](image2)
Figure B2. Nondimensional version of adjacent constant speed lines.