Wave attenuation by universal Venturi tubes: Finite difference predictions with analytical and experimental comparisons

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The wave attenuation performance of anechoically terminated universal Venturi tubes is investigated. For zero mean fluid flow the transmission loss is determined computationally in terms of a nonlinear time-domain technique; analytically, by combining the solutions for planar wave propagation in tapered ducts; and experimentally on an extended impedance tube setup with four fabricated configurations. Results from all three approaches are shown to compare well with each other. The numerical technique is also applied to examine the effect of compressible mean flow on the transmission loss. © 1996 Institute of Noise Control Engineering.

Primary subject classification: 37; Secondary subject classification: 75

1. INTRODUCTION

The working principles of the classical Venturi tube were discovered by Giovanni Battista Venturi in 1797, while it was first applied to practical flow metering by Clemens Herschel in 1887.12 Later, a number of modified Venturi tubes have been introduced, including the "Dall tube"3 and the "universal Venturi tube" (UVT). In comparison with the classical Herschel–Venturi configurations these designs offer several advantages related to flow measurements and space considerations, including magnified differential pressure drop, low permanent head loss, and short laying length. The flow characteristics of classical Venturi tubes,3 the Dall tube,5 and the UVT are well established due to their practical use as flow metering devices with low permanent losses. The noise attenuation performance of classical Venturi structures, however, has received little attention until recently.3,9

The objective of the present study is (1) to investigate the noise attenuation performance of anechoically terminated UVTs in the absence of mean flow, using analytical, computational, and experimental approaches; and (2) to examine the effect of bulk fluid motion on the acoustic performance of UVTs computationally. The working fluid is air at atmospheric conditions. For the frequency range of interest, and the dimensions considered in this study, the deviation from one-dimensional behavior, such as any curvature of wavefronts, is expected to be negligible. Thus, the numerical and analytical techniques applied here assume one-dimensional wave propagation. The computational approach employed to simulate unsteady, compressible flow through the UVTs is based on the finite difference time-domain method of Chapman, Novak, and Stein.10 Since the UVT design consists of a combination of conical and straight duct sections (see Fig. 1), the transmission loss may also be determined analytically by coupling the one-dimensional solutions available for the distinct segments of the configuration.11,12 Four UVTs with inlet/exit duct to throat area (contraction) ratios of \( m = 2, 4, 9, \) and 16, as shown in Fig. 1, are fabricated and tested in an anechoically terminated extended impedance tube setup. Inlet and exit duct diameters are chosen to match the dimensions of an existing acoustics test facility for the experimental work. The throat diameters and section lengths \( (d_f, d) \) are then dictated by the desired contraction ratios.

This study first discusses the computational approach employed to simulate the unsteady flow of a compressible fluid in ducts of varying cross section. Next, the analytical treatment of one-dimensional wave propagation in conical duct sections with zero mean flow is summarized, followed by a brief reference to the experimental setup. After a comparison of the no-flow acoustic results from these three approaches, the computational approach is applied to address the effects of bulk fluid motion. The study is then concluded with some final remarks.

2. COMPUTATIONAL APPROACH

The numerical technique is based on a finite difference approximation of the balance equations in the time domain for mass, momentum, and internal energy. For one-dimensional flow in ducts of variable cross section with neglected axial conduction, these governing equations may be expressed, respectively, as

\[
\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho AU) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t}(\rho AU) + \frac{\partial}{\partial x}(\rho AU^2) + \frac{\partial}{\partial x}(PA) - \tau_w \mathcal{P} = 0, \tag{2}
\]

\[
\frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x}(\rho AU e) + P \frac{\partial}{\partial x}(UA) - \tau_w \mathcal{P}U + q \mathcal{P} = 0, \tag{3}
\]

where \( U \) is the velocity, \( P \) is the pressure, \( \tau_w \) is the wall shear stress, \( \mathcal{P} \) is the perimeter, \( e \) is the internal energy, and \( q \) is the wall heat transfer rate. The ideal gas equation of state,

\[
P = (\gamma - 1)\rho e, \tag{4}
\]
Universal Venturi Tube

\[ \begin{array}{cccc}
\phi 4.859 & L_{total} \\
10.00 & l & d_T/2 & d_T/2 & L & 10.00 \\
\end{array} \]

Unit: cm

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<th>l</th>
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<td>1.812</td>
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<td>1.215</td>
<td>2.083</td>
<td>20.827</td>
<td>24.124</td>
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</table>

Fig. 1 – Geometry of the universal Venturi tube.

where \( \gamma \) is the ratio of specific heats, is used to relate the thermodynamic variables and close the system of equations. Equations (1)–(3) are discretized by employing the explicit finite difference method of Chapman, Novak, and Stein\(^{10}\) as discussed by Selamat, Dickey, and Novak,\(^{13}\) which may be referred to for a more detailed description of the technique. The staggered mesh used in the discretization divides a duct into cells with vector quantities located at nodal points, and scalar quantities at cell midpoints (Fig. 2). In locations where the flow is smooth, the computational approach is based upon a second-order numerical scheme. However, in regions where the solution is changing rapidly enough to cause nonphysical oscillations in a higher-order algorithm, such as in a shock wavefront, the technique reverts to a first-order approximation to suppress this spurious behavior. To match the assumptions of inviscid, adiabatic flow used in the analytical approach, \( \tau_w \) and \( q \) are set to zero in the results to follow.

3. ANALYTICAL APPROACH AND EXPERIMENTS

Consider one-dimensional wave propagation in a duct with gradually varying cross section and zero mean flow: the linearized balance equations for mass and momentum, coupled with the isentropic equation of state leads to the well-known Webster horn equation\(^{14-16}\)

\[
\frac{\partial^2}{\partial x^2} + \frac{1}{4A^2} \left( \frac{dA^2}{dx} - 2A \frac{d^2A}{dx^2} - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \right) A \frac{\partial p}{\partial x} = 0,
\]

where \( x \) is the axial coordinate, \( A \) is the cross-sectional area, \( c_0 \) is the mean speed of sound, \( t \) is time, and \( p \) is the acoustic pressure. In view of harmonic disturbances and separation of variables, the acoustic pressure and velocity in a conical duct may be expressed as

\[
p = A^{-1/2} [ C_+ e^{-ikx} + C_- e^{ikx} ] e^{i\omega t},
\]

\[
u = \frac{A^{-1/2}}{\rho_0} \left[ \begin{array}{c}
\frac{1}{r} \frac{dr}{dx} + ik \end{array} \right] e^{-ikx}
+ \left[ \begin{array}{c}
\frac{1}{r} \frac{dr}{dx} - ik \end{array} \right] e^{ikx} e^{i\omega t},
\]

where \( C_+ \) and \( C_- \) are complex amplitude constants for the positive and negative traveling waves, respectively, \( i \) is the imaginary unit, \( k = \omega/c_0 \) is the wave number, \( \omega = 2\pi f \) is the angular frequency, \( f \) is the frequency, \( \rho_0 \) is the mean fluid density, and \( r \) is the duct radius. The relationships between the pressure and velocity oscillations across a single conical section of length \( \ell / \text{k} \) may then be written in terms of \( \mathcal{R} = r_1 / \text{tan} \theta \), \( \theta \) being the divergence half-angle (see Fig. 3), as

\[
\begin{bmatrix}
p_{x=0} \\
u_{x=0}
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
p_{x=\ell / \text{k}} \\
u_{x=\ell / \text{k}}
\end{bmatrix},
\]

where

\[
T_{11} = \frac{r_2}{r_1} \cos \kappa \mathcal{R} - \frac{1}{k \mathcal{R}} \sin \kappa \mathcal{R},
\]

\[
T_{12} = i \frac{r_2}{r_1} \rho_0 c_0 \sin \kappa \mathcal{R},
\]

Fig. 2 – Variable centering for a computational cell in numerical technique.

\[ T_{21} = -\frac{i}{\rho_0 c_0} \left( \frac{r_2}{r_1} + \frac{1}{k^2 r_1^2} \right) \sin k' + \frac{1}{2k^2 r_1^2} \cos k' \],

\[ T_{22} = \frac{r_2}{r_1^2} \left( \cos k' + \frac{1}{k^2 r_1^2} \sin k' \right). \] (9)

The results expressed by Eqs. (8) and (9) are similar to Eq. (2.144a) of Munjal,\(^{17}\) with the exception that Munjal employs acoustic mass flow rate (\(\rho_0 u\)), rather than acoustic velocity (\(u\)) of the present study. A discussion of the effect of wavefront bending on this formulation can be found in Benade.\(^{18}\) Note that in the limit as \(r_2/r_1 \rightarrow 1\), \(\tan \theta \rightarrow 1\), and \(\Gamma \rightarrow \infty\), Therefore, the terms in Eq. (9) involving \(1/\Gamma^2\) and \(1/\Gamma^4\) vanish, and Eqs. (8) and (9) reduce, as expected, to

\[ \begin{bmatrix} p_{x=0} \\ u_{x=0} \end{bmatrix} = \begin{bmatrix} \cos k' & i \rho_0 c_0 \sin k' \\ -i \rho_0 c_0 \cos k' & \cos k' \end{bmatrix} \begin{bmatrix} p_{x=r} \\ u_{x=r} \end{bmatrix}. \] (10)

which is the well-known relationship for a straight pipe.

Transmission loss for an anechoically terminated silencer is defined by

\[ TL = 20 \log_{10} \left| \frac{p_{+,in}}{p_{+,tr}} \right|, \] (11)

where the subscripts \(\text{in}\) and \(\text{tr}\) refer to incident and transmitted components, respectively. In terms of the relevant transmission matrices, the transmission loss for an anechoically terminated UVT may be expressed as

\[ TL = 20 \log_{10} \left| \frac{1}{2} \left[ \begin{bmatrix} 1 & \rho_0 c_0 \end{bmatrix} T_1 T_2 T_3 T_4 \begin{bmatrix} 1 \\ \rho_0 c_0 \end{bmatrix} \right] \right|, \] (12)

where \(T_i\) represents the transmission matrix of the \(i\)-th conical Venturi section, numbered consecutively from the source to the anechoic termination. Note that Eq. (12) provides a clear and explicit relationship between the transfer matrices and the transmission loss. In view of Eq. (12), it may readily be shown, for example, by combining with Eq. (10) for the case of a straight pipe limit

\[ TL = 20 \log_{10} (\cos k' + i \sin k') = 0, \] (13)

as expected.

The four UVT geometries indicated in Fig. 1 have been fabricated and tested in an anechoically terminated extended impedance tube setup. The details of the experimental setup have been reported elsewhere.\(^{15}\) The transmission loss in the computational and experimental approaches is determined using a two-microphone technique.\(^{16}\)

4. COMPARISON OF RESULTS FOR NO-FLOW CASE

For the Venturi tubes of Fig. 1 with straight inlet/exit duct to throat area ratios \((m)\) of 2, 4, 9, and 16, Fig. 4 compares the transmission loss determined by the foregoing three approaches. The predictions from the computational approach agree well with the analytical results, as expected. The predictions also appear, in general, to agree favorably with the experimental transmission losses. All configurations exhibit an initial maximum followed by a relatively weaker one. The increase in noise attenuation with increasing \(m\) is expected since the duct to throat cross-sectional area change dominates the amount of reflection from the element. The observed broadband attenuation and its increase with \(m\) resemble the behavior of the limiting simpler configurations, such as contraction and expansion chambers, as elaborated in Selamet et al.\(^{2}\) Experimental results in Fig. 4 show, for the frequency range of 400–800 Hz, small but persistent deviations, which may be attributed to slight imperfections in the anechoic termination leading to some length-controlled reflections.

5. EFFECT OF MEAN FLOW

The computational approach has been employed to investigate the acoustic performance of UVTs carrying a compressible mean flow. The analysis assumes inviscid and
one-dimensional flow which neglects the effects of viscous dissipation and flow separation. At moderate flows these effects are expected to alter the transmission loss only marginally. Since the UVT configurations with \( m = 9 \) and \( m = 16 \) represent rather impractical flow elements, this part of the study concentrates on the geometries with \( m = 2 \) and \( m = 4 \). The transmission loss of each configuration is determined at a number of inlet Mach numbers \( M = [U/c_0]_{inlet} \) for source locations upstream and downstream of the UVT.

The computational results for the \( m = 2 \) geometry with upstream and downstream source locations are depicted in Figs. 5 and 6, respectively. The maximum main duct Mach number \( (M = 0.25) \) corresponds to a throat Mach number \( (M_t) \) of approximately 0.6. The overall broadband behavior remains somewhat similar to the no-flow case, and, for \( M \leq 0.10 \ (M_t \leq 0.2) \), the no-flow limit appears to yield a reasonable estimate. However, the transmission loss tends to differ substantially at higher flow rates, especially for the downstream source. Both configurations show an increase

![Graph 1](image1.png)

**Fig. 4** - Transmission loss for universal Venturi tube from analytical, computational, and experimental approaches.

![Graph 2](image2.png)

**Fig. 5** - Predicted transmission loss for universal Venturi tube with mean fluid flow; \( m = 2 \), upstream source.
in maximum transmission loss as the mean flow increases. With an upstream source, the location of the first maxima shifts toward lower frequencies, and transmission loss at the higher frequencies is reduced below the zero-flow value. For the downstream source, the first maxima shifts toward higher frequencies and the transmission loss increases with the mean flow at all frequencies considered.

Figures 7 and 8 show the computational results for the $m=4$ geometry with upstream and downstream source locations, respectively. For this UVT, $M=0.10$ is the maximum flow rate considered, corresponding to $M_r=0.45$. The basic effects of flow are similar to the $m=2$ results, with an increase in the maximum transmission loss and, beyond this maxima, the upstream and downstream source cases exhibit different trends. Contrary to the $m=2$ results, the location of the initial maxima shifts toward lower frequencies for a downstream source. At $M=0.05$ ($M_r=0.2$), and the frequency range considered, the transmission loss values are approximated reasonably well by the zero mean flow predictions.
6. CONCLUDING REMARKS

The present study has investigated the noise attenuation performance of four anechoically terminated universal Venturi tubes in terms of computational, analytical, and experimental approaches. With zero mean fluid flow, the computational predictions are in good agreement with the analytical method and the experimental data. As expected, introduction of mean fluid flow may affect the acoustic performance of the UVT, particularly at high flow rates. For the $m=2$ and $m=4$ geometries, and the frequency range considered here ($kr_{max} \approx 0.7$), the analytical (no-flow) results provide reasonable estimates of the transmission loss behavior for throat Mach numbers less than approximately 0.2.

When substantial bulk fluid motion is involved in addition to the oscillations, the noise attenuation performance needs to be evaluated in comparison with the flow efficiency, since the two often conflict with each other. Decreasing the throat to duct cross-sectional area ratio increases the sound attenuation at the expense of deteriorating flow efficiency. Thus the magnitude of noise suppression needs to be weighed against the flow efficiency, particularly at high flow rates, leading to a compromise. For further assessment of UVTs, the attenuation magnitudes obtained with acceptable flow losses should also be compared with those of alternative configurations with a comparable flow restriction.

7. ACKNOWLEDGMENT

An earlier report on this work was presented at Internoise 95, 1995.

8. REFERENCES