Circular concentric Helmholtz resonators\textsuperscript{a)}

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The effect of specific cavity dimensions of circular concentric Helmholtz resonators is investigated theoretically, computationally, and experimentally. Three analytical models are employed in this study: (1) A two-dimensional model developed to account for the nonplanar wave propagation in both the neck and the cavity; (2) a one-dimensional solution developed for the limit of small cavity length-to-diameter ratio, $l/d$, representing a radial propagation in the cavity; and (3) a one-dimensional closed-form solution for configurations with large $l/d$ ratios which considers purely axial wave propagation in the neck and the cavity. For low and high $l/d$, the resonance frequencies determined from the two-dimensional approach are shown to match the one-dimensional predictions. For cavity volumes with $l/d>0.1$, the resonance frequencies predicted by combining Ingard’s end correction with one-dimensional axial wave propagation are also shown to agree closely with the results of the two-dimensional model. The results from the analytical methods are then compared with the numerical predictions from a three-dimensional boundary element method and with experiments. Finally, these approaches are employed to determine the wave suppression performance of circular Helmholtz resonators in the frequency domain. © 1997 Acoustical Society of America. [S0001-4966(97)05312-5]

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INTRODUCTION

Helmholtz resonators, which consist of a volume communicating through an orifice or neck to some external excitation, produce narrow bands of high wave attenuation. The classical approach in modeling these resonators is to neglect the spatial distribution leading to an equivalent spring–mass system where the mass of air in the neck, $m = \rho_0 A_c l_c$, is driven by an external force and the volume acts as a spring with stiffness $s = \rho_0 c^2 A_c^2/V$, $A_c$ and $l_c$ being the neck area and length, respectively, and $V$ the resonator volume (Rayleigh, 1945, Kinsler et al., 1982, Chap. 10). For this one degree of freedom system, $\omega_0 = \sqrt{s/m}$, leading to a single resonance frequency of $f_c = (c_0/2\pi) \sqrt{A_c/l_c V}$, which is a function of the cavity volume, but independent of the volume dimensions. Experimental observations, however, have deviated from this frequency, which is attributed to the motion of some additional mass on both sides of the neck. To improve the accuracy, the neck length is usually “corrected” by adding a term for each end in order to account for this fluid motion, thereby modifying $l_c$ in the foregoing expression for $f_c$ by $l'_c = l_c + \delta_e + \delta_a$. A number of analytical treatments based on somewhat simplified physics have been introduced to develop these end correction factors. Rayleigh (1945) derived a length correction for resonators mounted in a baffle. Ingard (1953) developed an end correction to account for multidimensional wave propagation excited at the area discontinuity from the neck to the volume, but it is only effective for a limited range of geometries when used with the classical model above. In their extensive work, Miles (1971) and Miles and Lee (1975) investigated the Helmholtz resonance behavior of harbors by employing the electrical analogy; the former treats the simple shapes including circular and rectangular harbors with constant depth, the latter develops analytical approaches for irregular geometries with variable depth. Alster (1972) extended the spring-mass analogy by including the mass of the spring and incorporating a spring with varying stiffness. His experimental results for a number of different resonator shapes showed a significant improvement over the lumped model. Several other simple end corrections are also listed by Chuka (1973). To predict the resonance frequencies, Tang and Sirignano (1973) assumed one-dimensional wave propagation in both the resonator neck and cavity volume. Panton and Miller (1975) developed a relationship for the resonance frequency which matched experimental results well when used with Ingard’s end correction. Their effort was based on a one-dimensional wave motion in the cavity alone and a spatially lumped short neck length. Monkewitz and Nguyen-Vo (1985) studied nonplanar effects in two-dimensional resonators with a semicylindrical cavity and three-dimensional resonators with a hemispherical cavity. By asymptotically matching the solutions of the linearized inviscid equations in terms of low-frequency expansions in the exterior, neck, and cavity, they proposed volume and length corrections for these configurations. Selamet et al. (1993, 1995a) developed an expression for the transmission loss of a Helmholtz resonator with wave

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motion. They also provided experimental results illustrating that the relationship for resonance frequency given by Tang and Sirignano (without end correction factors) worked well for volumes with large length-to-diameter ratio, while showing some deviation at low \( l_v/d_v \) ratios (hereafter, \( l/d \), for brevity). In a preliminary investigation, Selamet et al. (1994a) studied the effect of multidimensional propagation and showed deviations in resonance frequency from the lumped parameter analysis, particularly at low \( l/d \) ratios. Recently, Chanaud (1994) developed a relationship for the resonance frequency of configurations with a rectangular parallelepiped cavity volume. Finally, by curve fitting finite element results, Sahasrabudhe et al. (1995) provided polynomial expressions for the end correction as a function of expansion ratio and frequency.

The present study considers the circular concentric Helmholtz configurations with constant cavity volume and neck length, as shown in Fig. 1. The objective is to investigate, as a function of the \( l/d \) ratio: (1) the discrepancy in the resonance frequency from classical approaches, and (2) the acoustic attenuation behavior. To estimate the end correction accurately at the neck–volume interface, a two-dimensional axisymmetric analytical model is developed for these resonators. The study also employs two different one-dimensional analytical solutions based on (1) radial propagation for small \( l/d \) ratios, and (2) axial propagation for large \( l/d \) ratios, to be referred hereafter as one-dimensional radial and axial solutions, respectively. Both closed-form solutions are used in the study to examine their corresponding limits. The pressure field inside the resonators is determined by the boundary element method and used to evaluate the degree of multidimensional propagation. The analytical predictions for the acoustic attenuation of the resonators are then compared with the computations and the experiments for the case of zero mean flow.

Following the Introduction, Sec. I describes the one-dimensional methods, Sec. II the two-dimensional analytical approach, Sec. III the boundary element method, and Sec. IV the experimental setup. The results from the analytical approaches and the boundary element method are compared with experiments and used to evaluate the end corrections in Sec. V. The study is concluded with final remarks in Sec. VI.

I. ONE-DIMENSIONAL ANALYTICAL APPROACHES

The generation and propagation of multidimensional waves in the resonator volume is clearly dependent on the relative magnitudes of the wavelength, the neck and duct diameters and the volume dimensions. Provided that the incident wave from the neck to the volume is planar, there are two limiting configurations for which the wave propagation in the volume can be considered one-dimensional, thereby allowing a relatively simple closed-form solution for transmission loss. This section presents the expressions for the wave attenuation properties for these one-dimensional limits.

A. Radial propagation limit

The discrepancy between the analytical results and the experiment is largest for \( l/d \) of the order 0.1 to 1.0. An accurate prediction of the transmission loss and resonance frequency in this region then requires a multidimensional analysis. For smaller \( l/d \) ratios as the volume approaches a “pancake” geometry, a one-dimensional solution in the radial direction becomes possible as discussed next.

Consider planar propagation in the main duct and connector and purely radial wave motion in the cavity volume. For circularly symmetric propagation of acoustic waves in a hollow disk of constant width, the linearized, inviscid wave equation may be written as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}.
\]

Introducing the harmonic dependence

\[
p(r,t) = P(r)e^{i\omega t},
\]

and using the separation of variables, the solution for complex pressure amplitude may be determined as

\[
P(r) = C_1 H_{l_v}^{(1)}(kr) + C_2 H_{l_v}^{(2)}(kr),
\]

where \( C_1 \) and \( C_2 \) are complex constants related to the amplitude of oscillation for the inward and outward traveling waves, respectively, and \( H_{l_v}^{(j)} \) is the Hankel function (the Bessel function of the third kind) of order \( \beta \) and type \( \eta \). As a function of the latter variable, \( \eta \), Hankel functions may readily be expressed as \( H_{l_v}^{(1)} = J + i Y \) and \( H_{l_v}^{(2)} = J - i Y \) with \( J \) and \( Y \) being the Bessel functions of the first and second kind, respectively. The momentum equation

\[
\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho_0} \nabla p
\]

combined with the harmonic relationship

\[
u(r,t) = U(r)e^{i\omega t}
\]

and Eqs. (2) and (3) yields the complex velocity amplitude as

\[
U(r) = -\frac{i}{\rho_0 c_0} [C_1 H_{l_v}^{(1)}(kr) + C_2 H_{l_v}^{(2)}(kr)].
\]
At the intersection of the neck and cavity, a constant pressure cylindrical junction is assumed, where the boundary conditions of continuity of pressure and conservation of flow volume are applied. To satisfy these requirements, the pressure and flow volume at \( r=r_c \) are matched with those at the cavity end of the neck. Furthermore, the rigid wall requires that \( u=0 \) at \( r=r_e \). Combining the foregoing relationships with those for planar propagation in the main duct and neck yields the transmission loss for a Helmholtz resonator in the low \( l/d \) limit as

\[
\text{TL} = 10 \log_{10} \left| 1 + \frac{A_c}{2A_p} \left[ 1 + iX \tan kl_c \right]^2 \right|^2, \quad (7)
\]

where

\[
X = \frac{Z_p}{\rho_0 c_0} \left( \frac{r_c}{2l_v} \right) \quad (8)
\]
is introduced for convenience, and

\[
Z_p = \frac{p|_{r=r_e}}{u|_{r=r_e}} = \frac{i\rho_0 c_0}{H^1_0(kr_c) - \frac{H^1_0(kr_0)}{H^2_0(kr_0)}} \left( \frac{H^1_0(kr_0)}{H^2_0(kr_0)} \right) \quad (9)
\]
is the volume impedance at \( r=r_c \).

**B. Axial propagation limit**

The effect of nonplanar wave propagation is expected to diminish as the cavity diameter approaches that of the neck, which is equivalent to increasing the \( l/d \) ratio of the volume for a specified volume. Considering only one-dimensional propagation in the axial direction in the neck and cavity volume leads to the closed-form relationship

\[
\text{TL} = 10 \log_{10} \left[ 1 + \frac{A_c}{A_p} \left( \frac{\tan kl_c + (A_v/A_c) \tan kl_v}{1 - (A_v/A_c)\tan kl_c \tan kl_v} \right)^2 \right] \quad (10)
\]

for transmission loss (Selamet et al., 1993, 1995a). The denominator of Eq. (10) provides a relationship for the resonance frequency as

\[
\tan kl_c \tan kl_v = \frac{A_c}{A_v}, \quad (11)
\]

(the form derived by Tang and Sirignano, 1973) which implies that the frequency is a function of the cavity dimensions.

**II. TWO-DIMENSIONAL ANALYTICAL APPROACH**

Nonplanar wave propagation in the vicinity of duct discontinuities has been studied by Miles (1944, 1946, 1948). Following his works, the present study considers axisymmetric wave propagation in concentric circular ducts and develops a two-dimensional analytical solution to account for the wave motion in the resonator neck and volume in terms of the resulting pressure waves. For propagation in a circular and concentric configuration, the solution to the linearized, inviscid wave equation

\[
\nabla^2 p = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} \quad (12)
\]
can be written (Munjal, 1987), in view of \( p(r, x, t) = P(r, x)e^{i\omega t} \), as a combination of planar and radial waves as

\[
P_A(r, x) = A_0 e^{-ikx} + \sum_{n=1}^{\infty} A_n J_0(\gamma_{j,0n}r) e^{ik_{j,0n}x} \quad (13)
\]

for a wave traveling in the positive \( x \) direction, and

\[
P_B(r, x) = B_0 e^{ikx} + \sum_{n=1}^{\infty} B_n J_0(\gamma_{j,0n}r) e^{-ik_{j,0n}x} \quad (14)
\]

for a wave traveling in the negative \( x \) direction. Here, \( P \) is the complex amplitude of \( p, J_0 \) is the Bessel function of the first kind and order zero, \( k = \omega/c_0 \) is the planar wave number, and \( k_{j,0n} \) is the wave number in the \( x \) direction given by

\[
k_{j,0n} = \sqrt{k^2 - \gamma_{j,0n}^2}, \quad (15)
\]

where \( \gamma_{j,0n} = \alpha_{0n}/r_j \) is the radial wave number in a pipe of radius \( r_j \), with \( \alpha_{0n} \) being the roots of the Bessel function \( J_0(\alpha_{0n}) = 0 \). In terms of the momentum equation,

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \nabla p, \quad (16)
\]

the velocities of these waves can be obtained as

\[
U_A(r, x) = \frac{1}{\rho_0 c_0} A_0 e^{-ikx} - \frac{1}{\rho_0 \omega} \sum_{n=1}^{\infty} A_n k_{j,0n} J_0(\gamma_{j,0n}r) e^{ik_{j,0n}x}, \quad (17)
\]

\[
U_B(r, x) = \frac{1}{\rho_0 c_0} B_0 e^{ikx} + \frac{1}{\rho_0 \omega} \sum_{n=1}^{\infty} B_n k_{j,0n} J_0(\gamma_{j,0n}r) e^{-ik_{j,0n}x}. \quad (18)
\]

Driving the resonator via a piston with an oscillating velocity amplitude of \( U_p \) allows the interface between the neck and volume to be isolated. For the piston-excited resonator shown in Fig. 2, matching the velocity boundary condition at the piston

\[
U_p e^{i\omega t} \big|_{x=0} = (u_A + u_B) |_{x=0}, \quad (19)
\]
gives \( s=0, 1, \ldots \) \( \infty \) equations; for \( s=0, \)

\[
U_p = \frac{1}{\rho_0 c_0} (A_0 - B_0), \quad (20)
\]

and for \( s=1, 2, \ldots, \infty, \)

\[
0 = A_s - B_s. \quad (21)
\]

At the expansion from the neck to the volume, \( x_c = l_c \), the pressure boundary condition
Similarly, the velocity boundary conditions

\( (P_A + P_B)|_{r = r_c} = (P_C + P_D)|_{r = r_c} \), for \( 0 \leq r \leq r_c \)  

(22)

(Miles, 1944) gives, for \( s = 0 \),

\[
A_0 \left( \frac{r_c^2}{2} \right) e^{-ikl_c} + B_0 \left( \frac{r_c^2}{2} \right) e^{ikl_c} = C_0 \left( \frac{r_c^2}{2} \right) + D_0 \left( \frac{r_c^2}{2} \right) + \sum_{n=1}^{\infty} C_n \left( \frac{r_c^2}{2} \right) e^{-ikl_c} + \sum_{n=1}^{\infty} D_n \left( \frac{r_c^2}{2} \right) e^{ikl_c},
\]

(23)

and for \( s = 1, 2, \ldots, \infty \),

\[
A_s e^{ikc} \left[ \frac{r_c^2}{2} \right] J_2^2(\gamma r_c) + B_s e^{-ikc} \left[ \frac{r_c^2}{2} \right] J_2^2(\gamma r_c) = \sum_{n=1}^{\infty} C_n \left[ \frac{r_c^2}{2} \right] \frac{J_2^2(\gamma r_c)}{\gamma^2 0n - \gamma^2 c 0s} \gamma^2 0n - \gamma^2 c 0s \gamma^2 0n - \gamma^2 c 0s} \gamma^2 0n - \gamma^2 c 0s}
\]

(24)

Similarly, the velocity boundary conditions

\( (u_{xA} + u_{xB})|_{r = r_c} = (u_{xC} + u_{xD})|_{r = r_c} \), for \( 0 \leq r \leq r_c \)  

(25)

\( (u_{xA} + u_{xD})|_{r = r_c} = 0 \), for \( r_c \leq r \leq r_v \),  

give, for \( s = 0 \),

\[
A_0 v^2 e^{-ikl} - B_0 v^2 e^{ikl} = C_0 v^2 - D_0 v^2,
\]

(27)

and for \( s = 1, 2, \ldots, \infty \),

\[
kA_0 \left[ \frac{r_c^2}{2} \right] J_1^2(\gamma r_c) e^{-ikl_c} - \sum_{n=1}^{\infty} A_n k_{c,n} \left[ \frac{r_c^2}{2} \right] J_1^2(\gamma r_c) e^{-ikl_c} - kB_0 \left[ \frac{r_c^2}{2} \right] e^{ikl_c} - \sum_{n=1}^{\infty} B_n k_{c,n} \left[ \frac{r_c^2}{2} \right] J_1^2(\gamma r_c) e^{-ikl_c} = -k v_{c,0} C_s \left[ \frac{r_v^2}{2} \right] J_0^2(\gamma r_v) + k v_{c,0} D_s \left[ \frac{r_v^2}{2} \right] J_0^2(\gamma r_v).
\]

(28)

For the reflection from the rigid wall at the end of the volume, Eq. (20) with \( U_p = 0 \) evaluated at \( x_v = l_v \) gives, for \( s = 0 \),

\[
0 = C_0 e^{-ikl} - D_0 e^{ikl},
\]

(29)

and for \( s = 1, 2, \ldots, \infty \), Eq. (21) gives

\[
0 = C_s e^{-ikl} - D_s e^{ikl},
\]

(30)

Equations (20), (21), (23), (24), and (27)–(30) provide a set of simultaneous equations to determine the pressure amplitudes in the neck and volume \( A_n, B_n, C_n, \) and \( D_n \) \( (n = 0, 1, \ldots) \) (Radavich, 1995; Selamet and Radavich, 1995b, 1995c). Higher-order radial terms have a diminishing effect on the solution, thereby allowing the truncation of the infinite series to a finite number of terms sufficient to provide an accurate solution for the pressure variation in the resonator. In this closed system, the resonance occurs when the pressure or the resistance on the piston is a minimum (Kinsler et al., 1982, Chap. 9). This minimum pressure at the piston can be evaluated by substituting \( A_n \) and \( B_n \) into Eqs. (13) and (14).

Figure 3 provides a comparison of the resonance frequency versus the \( l/d \) ratio predicted by the one-dimensional methods and the two-dimensional analytical approach. There is a good agreement between the one-dimensional and the
two-dimensional methods at the extremes of $l/d$ where the volume is either very short with a large diameter leading to a one-dimensional radial wave or very long and narrow producing an axial planar wave. In between, however, for $l/d$ ratios of approximately 0.1 to 3, hereafter to be referred as the mid-region, there is a considerable difference between the one- and two-dimensional methods with deviations reaching about 8%. The general trend in the resonance frequency at low and high $l/d$ for the present concentric circular configurations is in qualitative agreement with the analysis of Chanaud (1994) on square-faced parallelepiped cavities, as expected.

The transmission loss of the foregoing two-dimensional resonator can be determined on an impedance tube setup as shown in Fig. 4. The complex interface between the impedance tube and the resonator neck where the two circular tubes come together is an obstacle for the development of an exact two- or three-dimensional relationship. Therefore, only one-dimensional waves are assumed to propagate in the impedance tube in order to isolate the multidimensional effects of the expansion. This approximation requires the presence of planar waves at the impedance tube–neck interface as shown in Fig. 4. Here the planar input wave $E$ produces a planar wave $A$ that travels up the resonator neck. At the expansion from the neck to the volume, the sudden area discontinuity excites the two-dimensional waves $C$ and $D$ in the volume as in Fig. 2. The boundary conditions at the neck–cavity interface require a two-dimensional wave $B$, while the boundary conditions at the neck–impedance tube interface require that $B$ be one-dimensional. Both conditions are satisfied by assuming that $B$ is two-dimensional at the neck–cavity volume interface, and that the radial modes decay sufficiently over the length of the neck before $B$ reaches the impedance tube junction. This assumption is justified for frequencies well below the cutoff frequency for the neck. For the impedance-tube-mounted resonator, the equations for the reflection at the end of the resonator volume, Eqs. (29) and (30), remain unchanged. For the expansion from the resonator neck into the volume, Eqs. (23), (24), (27), and (28) are used with $A_n$ ($n = 1, 2, ..., \infty$) set to zero, while retaining only the planar $A_0$ term. At the impedance tube interface, the pressure boundary condition gives

$$A_0 + B_0 = (E + F) = G \quad (31)$$

and the velocity boundary condition gives

$$(E - F) r_p^2 = (A_0 - B_0) r_p^2 + G r_p^2 \quad (32)$$

leading to a set of simultaneous equations if the input magnitude $E$ is specified. The transmitted wave $G$ can then be determined allowing for the calculation of the transmission loss across the resonator as

$$TL = 20 \log_{10} \frac{|E|}{|G|}. \quad (33)$$

For a typical geometry within the mid-region of Fig. 3, for example $l/d=1.0$, Fig. 5 provides a transmission loss comparison between the axial one-dimensional method of Eq. (10) and the two-dimensional approach. The magnitude of deviation observed in this figure is large enough to lead to the misprediction of the primary transmission loss due to the narrow attenuation band of the Helmholtz resonator.

This analytical study illustrates the significance of the multidimensional effects in the vicinity of neck–volume interface. The remainder of the work concentrates then on the multidimensional physics in view of the three-dimensional...
FIG. 7. Pressure magnitude (Pa) contours for Helmholtz resonator with $l/d=0.01$.

FIG. 8. Pressure magnitude (Pa) contours for Helmholtz resonator with $l/d=0.1$. 
FIG. 9. Pressure magnitude (Pa) contours for Helmholtz resonator with $l/d = 1.0$.

FIG. 10. Pressure magnitude (Pa) contours for Helmholtz resonator with $l/d = 10$. 
computations and experiments conducted in an extended impedance tube facility. The next section provides a brief description of the three-dimensional computational approach based on the direct boundary element method.

III. BOUNDARY ELEMENT METHOD

To analyze the multidimensional effects at the neck-to-volume area transition further, a three-dimensional direct boundary element method is used. A detailed account of this method, which is based on the linearized, inviscid wave equation [see Eq. (12)], can be found in numerous sources (Rayleigh, 1945; Art. 293; Seybert et al., 1985; Soenarko and Seybert, 1991). The boundary element method was used to model the experimental apparatus: An oscillating velocity was input; an anechoic termination was implemented by setting the impedance of the termination equal to the characteristic impedance of the fluid, \( \rho c_0 \); and the two-microphone technique was used to calculate the transmission loss. For the present investigation, the boundary element method was implemented using isoparametric quadrilateral and triangular elements. To ensure accuracy, a fine mesh spacing of less than 2.5 cm was maintained for all models. The largest mesh size of 2978 nodes occurred for a small cavity \( l/d \) of 0.01, which greatly increases the surface area to be discretized. As a result of this large surface area, no configuration below \( l/d \) of 0.01 was modeled with the boundary element method. A sample mesh for an \( l/d \) of 1.0 is shown in Fig. 6.

In order to investigate the differences between the one- and two-dimensional methods and determine the extent of nonplanar wave propagation, particularly in the mid-region of \( l/d \), pressure contours on the symmetry plane of the configurations were examined at their respective resonance frequencies using the boundary element method for \( l/d \) ratios of 0.01, 0.1, 1.0, and 10, as shown in Figs. 7 through 10. For all four configurations, some nonplanar wave bending is observed at the junction between the neck and impedance tube. Focusing on the interaction between the neck and the volume, at the low \( l/d \) of 0.01, Fig. 7 (\( f_r = 77 \) Hz) shows clearly one-dimensional radial propagation in the volume. Figure 8 (\( f_r = 90 \) Hz) illustrates the contours at a lesser extreme of the mid-region in Fig. 3 with \( l/d = 0.1 \). As the \( l/d \) ratio is increased to the \( l/d = 1.0 \) configuration of Fig. 9 (\( f_r = 91 \) Hz), multidimensional wave propagation becomes evident at the area transition between the neck and the volume. For the high \( l/d = 10 \) case in Fig. 10 (\( f_r = 72 \) Hz), some nonplanar bending is observed at the neck–volume interface, but the overall propagation is mostly planar. The extreme radial and axial configurations of \( l/d = 0.01 \) and 10 exhibit a marked pressure variation over the length of the volume. The two mid-region configurations of \( l/d = 0.1 \) and 1.0, however, show a rather small pressure variation in the volume. Thus the configurations in this region may be approximated by a lumped volume approach, provided the physics at the transitions is incorporated accurately. Examining the area transition for the \( l/d = 1.0 \) and \( l/d = 10 \) cases in Figs. 9 and 10 reveals that the multidimensional wave propagation is more pronounced for the larger area transition, as expected. In the one-dimensional axial model, it is assumed that the waves in the volume are planar immediately after the area transition and that the pressure and velocity suddenly change over this discontinuity when in reality there is a portion of the fluid in the volume that moves with the fluid in the neck. This effect has typically been accounted for in one-dimensional axial theory by adding a length correction to the neck which will be discussed in detail following a brief description of the experimental setup.

IV. EXPERIMENTS

The experimental apparatus consists of an extended impedance tube configuration, where the Helmholtz resonators are placed between a broad-frequency noise source and an anechoic termination. The two-microphone technique (Chung and Blaser, 1980; ASTM, 1990) is utilized to separate incident and reflected waves for calculation of the transmission loss across the element, with one pair of microphones placed before and another pair after the resonator. Although multidimensional waves are excited in the resonator volume, the selected impedance tube diameter ensures planar propagation at the microphones, with a cutoff frequency above 4 kHz for nonsymmetric modes. For further details of the experimental setup, refer to Selamet et al. (1994b). Eight Helmholtz resonators with circular concentric neck and cavity volumes, as described in Table I, were fabricated for the experimental study. The impedance tube diameter, the neck length and diameter, and the resonator volume \( (d_p, l_c, d_c, \text{ and } V) \) are fixed for all resonators, while the length-to-diameter ratio, \( l/d \), is varied. No resonator was built below \( l/d = 0.32 \) configuration due to fabrication difficulties for the set of parameters employed in the study.

V. RESULTS AND DISCUSSION

Numerous corrections for both ends of the resonator neck proposed by a number of investigators lack universal applicability since the expressions are derived (1) usually to match one set of experimental data; and (2) to be used in the classical lumped approach, which is known to deviate from experiments as the \( l/d \) ratio of the volume increases, due to the neglect of wave motion effects in the neck and cavity volume (Selamet et al., 1995a). Many of these end corrections also fail to incorporate the effect of expansion ratio from the resonator neck to the cavity, which are observed clearly in Fig. 3, and instead involve only the neck dimensions. An end correction that accounts for nonplanar wave propagation effects between the neck and volume was suggested by Ingard (1953) who modeled the neck as a piston oscillating into an expanded pipe of infinite length. His result for the end correction \( \delta \) may be expressed in the dimensionless form as

\[
\delta = 2 \sum_{n=1}^{\infty} \frac{1}{a_{0n}^2} \left[ J_1(\alpha_{0n} d_c/l_d) \right]^2 
\]

where \( J_0 \) and \( J_1 \) are the Bessel functions of the first kind, \( \alpha_{0n} = \gamma_{0n} r_p \), \( J_1(\alpha_{0n}) = 0 \), and \( l_c = l + \delta \). Equation (34) may also be approximated by a simple linear expression

\[
\delta \approx 0.85 \left[ \frac{d_c}{2} \right] \left[ 1 - 1.25 \frac{d_c}{l_c} \right]^{-1}
\]
for \(d_c/d_e<0.4\), which reduces to \(\delta=0.85(d_c/2)\) as \(d_c/d_e\) becomes negligible. Recall the end corrections suggested by Rayleigh for a pipe opening with an infinite flange (King, 1936; Rayleigh, 1945, Art. 307; Miles, 1948) as

\[
\frac{\pi}{4} \left( \frac{d_c}{2} \right) \delta < \frac{8}{3\pi} \left( \frac{d_c}{2} \right) \delta \,
\]

or \(0.78540<\delta/(d_c/2)<0.84883\). Thus Eq. (34) approaches Rayleigh's upper limit as \(d_c/d_e \to 0\), as illustrated by Eq. (35). By assuming a quadratic velocity profile over the opening, Rayleigh improved this correction further as \(\delta/(d_c/2)<0.82422\). Later, King (1936) determined the correction more accurately as \(\delta/(d_c/2)=0.82132\). At low \(l/d\), the expansion is large and the correction from Eq. (34) is at a maximum, which reduces the resonance frequency of the classical approach and yields corrected one-dimensional predictions closer to the two-dimensional results. With increasing \(l/d\) ratio, however, the correction approaches zero and the resonance frequency increases, which contradicts the results depicted in Fig. 3. Although Eq. (34) involves nonplanar wave propagation at the neck-cavity interface, its neglect of one-dimensional axial propagation leads to a discrepancy as the \(l/d\) ratio is increased. Combination of Eq. (34) with Eq. (11) allows for one-dimensional axial wave propagation throughout the resonator and approximates the multidimensional physics at the area contraction. This corrected one-dimensional axial and the two-dimensional analytical approaches are compared in Fig. 11. For \(l/d\) ratios greater than about 0.1, the two methods agree well. At \(l/d\) ratios less than about 0.1, the two-dimensional model predicts a decrease in the resonance frequency as the volume begins to resonate radially (the one-dimensional radial propagation begins to dominate), whereas the one-dimensional axial predictions remain nearly constant.

Experimental results as well as the boundary element method predictions for the configurations listed in Table I are also included in Fig. 11. For the geometries investigated, the boundary element method provides a slight improvement over the other methods. This is attributed to the ability of the boundary element method to incorporate the complicated geometry and multidimensional wave propagation at the junction between the impedance tube and the neck which was left uncorrected for the one- and two-dimensional analysis. Thus the three-dimensional computations take the so-called “radiation impedance” effect into account inherently on both the cavity and the impedance tube sides of the neck.

For \(l/d\) ratios of 0.32 and 1.59 in the mid-region of Fig. 11, Figs. 12 and 13 show significant improvements in the attenuation predictions of the one-dimensional axial method when the correction factor is used. For a large \(l/d\) ratio of 23.92, which shows only a small deviation between the one-dimensional approach and experiments in Fig. 11, Fig. 14 illustrates the diminishing effect of the correction Eq. (34) with a decreased expansion ratio. These three figures also exhibit only minor differences between the approximate two-dimensional and the corrected one-dimensional approaches, as expected in view of the fact that all three use similar assumptions for \(l/d\) ratios greater than about 0.1.
TABLE I. Helmholtz resonator geometry

<table>
<thead>
<tr>
<th>Resonator</th>
<th>$l_p$ (cm)</th>
<th>$d_p$ (cm)</th>
<th>$l_p/d_p$ or $d_p/l_p$</th>
<th>$d_p/d_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.423</td>
<td>26.081</td>
<td>0.32</td>
<td>6.45</td>
</tr>
<tr>
<td>2</td>
<td>15.865</td>
<td>19.004</td>
<td>0.83</td>
<td>4.70</td>
</tr>
<tr>
<td>3</td>
<td>24.420</td>
<td>15.319</td>
<td>1.59</td>
<td>3.79</td>
</tr>
<tr>
<td>4</td>
<td>35.281</td>
<td>12.743</td>
<td>2.77</td>
<td>3.15</td>
</tr>
<tr>
<td>5</td>
<td>55.550</td>
<td>10.155</td>
<td>5.47</td>
<td>2.51</td>
</tr>
<tr>
<td>6</td>
<td>71.653</td>
<td>8.941</td>
<td>8.01</td>
<td>2.21</td>
</tr>
<tr>
<td>7</td>
<td>96.012</td>
<td>7.727</td>
<td>12.43</td>
<td>1.91</td>
</tr>
<tr>
<td>8</td>
<td>148.365</td>
<td>6.210</td>
<td>23.92</td>
<td>1.54</td>
</tr>
</tbody>
</table>


dimensional method in these figures gives slightly improved results for the reasons already indicated. In Figs. 11–14, the difference between the measured and computed (3-D BEM) resonance frequencies is a mere 1 Hz, which may possibly be attributed to rather minor structural vibrations and other losses in the experimental setup. The magnitude difference in the transmission loss between the analytical predictions and the experiments at resonance may be attributed to the neglect of viscothermal losses in the analytical treatments. The effect of viscous dissipation is expected to increase at either extreme of the $l/d$ ratio, where the distance between the walls may become comparable to the boundary layer thickness of the oscillating air.

VI. CONCLUDING REMARKS

This study has shown the effect of both planar and non-planar wave propagation on the resonance frequency and the wave attenuation of concentric Helmholtz resonators. The one-dimensional models including wave propagation in radial or axial directions and the experiments demonstrate that the resonance frequency changes as the cavity dimensions are varied. Full three-dimensional computations with the boundary element method illustrate that the deviations between the one-dimensional approaches and the experiments for cavities with $l/d$ ratios of about 0.1 to 3 are mostly due to nonplanar wave propagation at the area discontinuities. A two-dimensional analytical approach was then introduced to examine the multidimensional wave propagation at the area discontinuity from the neck to the cavity volume for an axisymmetric resonator. The results for the transmission loss and the resonance frequency from the two-dimensional technique are found to agree well with those of the one-dimensional axial approach modified with Ingard’s end correction for length-to-diameter ratios greater than 0.1. These results deviate slightly from the experiments, as expected, because of the difference in the neck–tube interface between the two configurations.

The effect of mean flow across the orifice is currently being investigated, which cannot only alter the resonance frequencies but, under the proper flow conditions, can turn the resonator into a noise generator rather than a silencer. Experimental work by Panton (1990) demonstrates that this effect is heavily dependent on the geometry of the orifice, as well as the nature of the boundary layer as it passes over the resonator. An experimental study combined with 3-D computations is also under progress for the low $l/d$ range (less than 0.1) in combination with the effect of shear on the wave propagation in the same range.

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