Acoustic Attenuation Performance of Perforated Absorbing Silencers

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ABSTRACT

The acoustic attenuation performance of a single-pass, perforated concentric silencer filled with continuous strand fibers is investigated theoretically and experimentally. One-dimensional analytical and three-dimensional boundary element methods are employed to predict the acoustic attenuation in the absence of mean flow. Measured complex characteristic impedance and wave number are used to account for the wave propagation through absorbing fiber. The perforation impedance facing the fiber is also presented in terms of the complex characteristic impedance and wave number. The effects of perforate duct porosity and the fiber density are examined. Comparisons of predictions with the experiments illustrate the need for multidimensional analysis at higher frequencies, while the one-dimensional treatment provides a reasonable accuracy at lower frequencies, as expected. The study also shows a significant improvement in the acoustic attenuation of the silencer due to fiber absorption.

1. INTRODUCTION

The sound absorption characteristics of fiber materials are well established in the literature (Cofer et al., 1999). The recent improvements in their properties combined with their broadband acoustic dissipation characteristics make such materials potentially desirable for implementation in silencers. The use of fibers may prove particularly effective when their dissipative characteristics are combined with the reactive silencers, leading to hybrid configurations.

Craggs (1977) examined the expansion chambers lined with absorbing material using a finite element method. He has shown that (1) the absorbing material increases the magnitude and changes the shape of transmission loss, and (2) increasing the thickness of absorbing material reduces the number of domes and shifts the peak frequencies of transmission loss. Wang (1999) analyzed a single-pass perforated absorbing silencer in terms of a one-dimensional decoupled method. To account for the acoustic characteristics of absorbing material, Wang used complex characteristic impedance and wave number, which depend on tortuosity, Prandtl number, and porosity. While simulation results were presented for a variety of parameters, such results were not validated by experimental work. Empirical expressions of Sullivan and Crocker (1978) were used for the perforation impedance, which were originally developed for perforations in the absence of absorbing material. Thus, the effect of absorbing material on the perforation impedance was neglected in Wang’s work.

The understanding of material properties is essential in studying the behavior of absorbing silencers. Delany and Bazley (1970) suggested empirical expressions for the characteristic impedance and wave number for fibrous absorbing material as a function of frequency and flow resistance. They found that the flow resistance was determined by fiber size and bulk density. Recently, Song and Bolton (2000) estimated the characteristic impedance and wave number of porous material by using measured pressures and a transfer matrix. The elements of the transfer matrix were evaluated from a single microphone approach and then the reciprocity of the matrix was used to calculate the acoustic properties of absorbing material. The characteristic impedance and wave number estimated by the transfer matrix method agree with the empirical expressions of Delany and Bazley (1970). Their conclusion that the acoustic property of the material is independent of sample depth and termination condition is also adopted in the present study.

Absorbing materials are typically used in combination with perforated ducts or screens, resulting in an interaction between them. Acoustic characteristics of the porous layer facing perforations were investigated by Ingard and Bolt (1951), who considered the perforation as an addition of mass. Recently Kirby and Cummings (1998) extended this work by investigating two types of perforations, circular and louvered plates, with and without porous backing. They have concluded that (1) the porous material increases the perforation
impedance, and (2) the reactance term facing absorbing material has both real and imaginary components, with imaginary part of the reactance resulting in an increase in resistance of the total perforation impedance. They have suggested that single hole data may be applicable for multi-hole perforations, thereby neglecting the interactions between the holes.

The objective of the present study is to investigate theoretically and experimentally the acoustic performance of uniformly perforated absorbing silencers with different duct porosity and material density. The predictions from the one-dimensional decoupled approach and the three-dimensional boundary element method are compared to experimental results obtained with an extended impedance tube setup. The effect of absorbing material on the perforation impedance is considered using both real and imaginary values for the end correction. The study assumes that (1) the absorbing material is homogeneous, isotropic, and rigid frame; (2) there is no mean flow; (3) the thickness of the perforated duct is much smaller than the wavelength; and (4) the characteristic impedance and wave number are independent of the depth of porous material.

Following this Introduction, the theoretical approach is presented in Section 2. Section 3 compares the transmission loss between predictions and experiments, and examines the effect of duct porosity and material density on the overall acoustic performance of silencers. Section 4 concludes the study with final remarks.

2. THEORY

Figure 1 shows the geometry of a single-pass absorbing silencer considered in this study with perforated duct inner diameter $d_1$, wall thickness $wt$, porosity $\phi$, outer chamber inner diameter $d_2$, length $\ell$, perforate hole diameter $hd$, and absorbing fiber material density $\rho_f$.

![Figure 1. A single-pass perforated absorbing silencer.](image-url)

This section describes briefly the one-dimensional decoupled and three dimensional boundary element methods followed by the perforate impedance and acoustical properties of absorbing material used in the present study.

### 2.1 ONE-DIMENSIONAL DECOUPLED METHOD

Assuming harmonic planar wave propagation in both the center perforated duct and expansion chamber, the continuity and momentum equations yield, in the absence of mean flow (Wang, 1999),

$$\frac{d^2 p_1}{dx^2} + \alpha_1 p_1 + \alpha_2 p_2 = 0,$$

$$\frac{d^2 p_2}{dx^2} + \alpha_3 p_1 + \alpha_4 p_2 = 0,$$

where

$$\alpha_1 = k^2 - \frac{4i}{d_1 \zeta_p},$$

$$\alpha_2 = \frac{4i}{d_1 \zeta_p},$$

$$\alpha_3 = \frac{4d_1}{d_2^2 - d_1^2} \frac{\tilde{\rho}}{\rho_0} \frac{ik}{\zeta_p},$$

$$\alpha_4 = k^2 - \frac{4d_1}{d_2^2 - d_1^2} \frac{\tilde{\rho}}{\rho_0} \frac{ik}{\zeta_p},$$

and $\rho_0$ and $k$ denote, respectively, the density and the wave number in air, and $\tilde{\rho}$ and $\tilde{k}$ the complex dynamic density and the complex wave number in the absorbing material, and $\zeta_p$ the acoustic impedance of perforation. Equations (1) and (2) may be expressed as,

$$\begin{bmatrix}
y_1' \\
y_2' \\
y_3' \\
y_4'
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\alpha_1 & -\alpha_2 & 0 & 0 \\
-\alpha_3 & -\alpha_4 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{bmatrix},$$

where $y_1 = p_1$, $y_2 = p_2$, $y_3 = \frac{dp_1}{dx}$, $y_4 = \frac{dp_2}{dx}$, and $(\cdot)'$ indicates derivative with respect to $x$. By using the numerical decoupling approach, the acoustic pressure and particle velocity at the inlet ($x = 0$) and outlet ($x = \ell$) may be related by
\[
\begin{bmatrix}
    p_1(0) \\
    p_2(0) \\
    \rho_0 c_0 u_1(0) \\
    \rho_0 c_0 u_2(0)
\end{bmatrix} = [R] \begin{bmatrix}
    p_1(\ell) \\
    p_2(\ell) \\
    \rho_0 c_0 u_1(\ell) \\
    \rho_0 c_0 u_2(\ell)
\end{bmatrix},
\]  

(8)

For the outer (expansion) chamber, the boundary conditions at \( x = 0 \) and \( x = \ell \) may be written as

\[
\frac{p_2(0)}{u_2(0)} = i \rho \xi \cot(\xi \ell),
\]  

(9)

\[
\frac{p_2(\ell)}{u_2(\ell)} = -i \rho \xi \cot(\xi \ell),
\]  

(10)

where \( \xi = \omega / \xi \) is the complex speed of sound. Finally, combining Eqs. (8) — (10) yields

\[
\begin{bmatrix}
    p_1(x = 0) \\
    \rho_0 c_0 u_1(x = 0)
\end{bmatrix} = [T_{ij}] \begin{bmatrix}
    p_1(x = \ell) \\
    \rho_0 c_0 u_1(x = \ell)
\end{bmatrix},
\]  

(11)

which defines the transfer matrix elements, \( T_{ij} \).

Assuming a duct with constant cross-sectional area, transmission loss can then be calculated from the transfer matrix as follows:

\[
TL = 20 \log_{10} \left( \frac{1}{2} \left| T_{11} + T_{12} + T_{21} + T_{22} \right| \right).
\]  

(12)

2.2 THREE-DIMENSIONAL BOUNDARY ELEMENT METHOD

The wave propagation is governed by the Helmholtz equation in the perforated duct

\[
\nabla^2 p_1 + k^2 p_1 = 0,
\]  

(13)

and in the outer chamber

\[
\nabla^2 p_2 + \tilde{k}^2 p_2 = 0,
\]  

(14)

where symbols were defined in the preceding section. Integrating Eqs. (13) and (14) and discretizing the boundary surfaces into a number of elements yields (Ji and Selamet, 1999)

\[
\begin{bmatrix}
p_1^i \\
p_1^o \\
p_1^p
\end{bmatrix} = [T_i] \begin{bmatrix}
u_1^i \\
u_1^o \\
u_1^p
\end{bmatrix},
\]  

(15)

where \( u_1^i \) and \( u_1^o \) are normal outward particle velocity at the boundaries of the perforated duct and outer chamber, respectively, and superscripts \( i, o, \) and \( p \) denote inlet, outlet, and perforate. The boundary conditions at perforate may be expressed as,

\[
u_1^o = -u_1^p,
\]  

(17)

\[
p_1^p - p_2^p = \rho_0 c_0 \xi_p u_1^p,
\]  

(18)

where \( r_0 c_0 \) is the characteristic impedance of air and \( \xi_p \) the perforate acoustic impedance. Combining Eqs. (15) — (18) yields the transfer matrix of a silencer, defined by Eq. (11), and therefore the transmission loss in view of Eq. (12).

2.3 ACOUSTIC IMPEDANCE OF PERFORATES

In Eqs. (3) — (6) and (18), the perforate impedance \( \xi_p \) relates the acoustic pressures in the inner duct and the outer chamber at the interface. Sullivan and Crocker (1978) presented empirical expressions for perforate acoustic impedance considering hole interactions. For low velocities through the holes, the acoustic impedance is given by

\[
\xi_p = \frac{0.006 + ik(t_w + 0.75d_h)}{\phi},
\]  

(19)

where \( t_w \) is the duct wall thickness, \( d_h \) the perforate diameter, \( \phi \) the porosity. However, Eq. (19) has been developed in the absence of filling material. For perforations facing absorbing material, such an equation needs to be modified, in view of the work by Kirby and Cummings (1998), as

\[
\xi_p = \left[ 0.006 + ik \left( t_w + 0.375d_h \left( 1 + \frac{\rho \xi}{\rho_0 \xi_0} \frac{k}{\tilde{k}} \right) \right) \right] / \phi.
\]  

(20)

It is assumed that the interactions among perforates through absorbing material are the same as those through air. In comparison with Eq. (19), the use of complex values for the characteristic impedance \( \rho \xi \) and wave number \( \tilde{k} \) in Eq. (20) changes both real and imaginary parts of the perforation impedance. When the medium is air, \( \rho \xi / \rho_0 \xi_0 \) and \( \tilde{k} / k \) become unity, thereby reducing Eq. (20) to Eq. (19). Thus, Eqs. (19) and (20) have been employed in this study for silencers without and with filling material, respectively.
2.4 WAVE PROPAGATION IN ABSORBING MATERIAL

The absorption of acoustic waves in filling material is mainly due to viscous and thermal dissipation, which may be expressed in terms of complex characteristic impedance and wave number (Beranek, 1988). The imaginary part of the wave number accounts for the decay of the waves, which is called the attenuation constant.

Since the structure of absorbing material is complicated, the acoustic properties are often determined experimentally. Delany and Bazley (1970) presented empirical expressions for the complex characteristic impedance and wave number of an absorbing material as follows:

\[
\tilde{Z}_c = \left[ 1 + 0.0511 \left( \frac{f}{R} \right)^{-0.75} \right] + i \left[ -0.0768 \left( \frac{f}{R} \right)^{-0.73} \right],
\]

(21)

\[
\tilde{k} = \left[ 1 + 0.0858 \left( \frac{f}{R} \right)^{-0.70} \right] + i \left[ -0.1749 \left( \frac{f}{R} \right)^{-0.59} \right],
\]

(22)

where \( f \) denotes frequency and \( R \) the resistivity.

The present study uses texturized fiber glass roving. The texturization process separates 4000 filament roving strands of fiber glass essentially into individual filaments by turbulent air flow (Silentex™ process). The average diameter of the individual filaments in the roving strand is 24 \( \mu \)m. The degree to which the strands are separated into individual filaments affects both the complex wave number and the impedance of the absorbing material. A description of the physical and chemical properties of various absorptive materials and their relative durability in simulated automotive silencer operations is described by Huff (2001). The acoustic properties of the absorbing material used in the present study have been measured at Owens Corning laboratories and fitted by

\[
\tilde{Z}_c = \left[ 1 + 0.0855 \left( \frac{f}{R} \right)^{-0.754} \right] + i \left[ -0.0765 \left( \frac{f}{R} \right)^{-0.732} \right],
\]

(23)

\[
\tilde{k} = \left[ 1 + 0.1472 \left( \frac{f}{R} \right)^{-0.577} \right] + i \left[ -0.1734 \left( \frac{f}{R} \right)^{-0.595} \right],
\]

(24)

for \( \rho_0 = 1.1555 \text{ kg/m}^3 \). For the measurement technique as well as the acoustic characteristics of some other absorbing materials, including fiberglass board, shoddy, and cellulose, the reader is referred to Nice and Godfrey (2000). Similar to Eqs. (21) and (22) of Delany and Bazley (1970), Eqs. (23) and (24) depend on the flow resistivity and frequency. The flow resistivity \( R \) in Eqs. (23) and (24) can be determined experimentally as shown in Table 1 for two different bulk densities used in this study.

<table>
<thead>
<tr>
<th>Bulk density (g/L)</th>
<th>Flow resistivity (MKS Rayls/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4,896</td>
</tr>
<tr>
<td>200</td>
<td>17,378</td>
</tr>
</tbody>
</table>

3. RESULTS AND DISCUSSION

The present study considers a single-pass concentric silencer of length \( l = 25.72 \text{ cm} \) with a uniformly perforated duct and an outer chamber of diameters \( d_1 = 4.9 \text{ cm} \) and \( d_2 = 16.44 \text{ cm} \), respectively, perforate hole diameter \( d_h = 0.249 \text{ cm} \), wall thickness of perforated duct \( t_w = 0.09 \text{ cm} \), and two porosities: \( \phi = 2\% \) and \( 8\% \). The two microphone technique is used in experiments to measure the transmission loss. The experimental setup is described elsewhere (Selamet et al., 1994).

Figure 2. Experimental results; transmission loss of single-pass perforated absorbing silencer with 8% porosity.

Figure 2 shows the measured transmission loss for the empty versus filled (with \( \rho_f = 100 \text{ g/L} \) and 200 g/L absorbing material) silencer. All three configurations have \( \phi = 8\% \) porosity. Both silencers with absorbing material have significantly higher acoustic attenuation than the one with no filling above 250 Hz. The silencer with no filling has several broadband attenuation domes up to 2000 Hz, resembling the behavior of expansion chambers. The addition of absorbing material changes the acoustic behavior of the silencer drastically, by switching to a single broad peak. Increasing the density
of the absorbing material from $\rho_f = 100$ g/L to 200 g/L increases the peak transmission loss, as well as shifting its location to a lower frequency.

Figure 3. Transmission loss of single-pass perforated absorbing silencer with 8 % porosity and no filling material.

For a silencer with no filling, the one-dimensional analytical and the boundary element methods are compared with the experiments in Fig. 3. While the boundary element method shows a good agreement with experiments for the overall frequency range except around 2100 Hz, the one-dimensional predictions start to deviate from measurements at 1500 Hz and fails completely at higher frequencies. The inaccuracy of one-dimensional analysis above 1500 Hz is due to the neglect of higher order modes. Both the one-dimensional analytical and the boundary element methods employ Eq. (19) (Sullivan and Crocker, 1978) for the perforation impedance.

The predictions for two silencers with filling densities $\rho_f = 100$ g/L and 200 g/L are compared with experiments in Figs. 4 and 5, respectively. While the boundary element method predictions show a reasonable agreement with experiments in the frequency range of interest, the one-dimensional analysis approach appears to capture the trends until the peaks around 1500 Hz and 800 Hz in Figs. 4 and 5, respectively, and deviate significantly above these values. Thus the one-dimensional method starts failing at lower frequencies with increasing density.

Figure 6. Experimental results; transmission loss of single-pass perforated absorbing silencer with 2 % porosity.
To examine the effect of perforate porosity, another duct with $\phi = 2\%$ is fabricated, while keeping the rest of the geometry the same. The experimental results for this porosity are shown in Fig. 6 for empty versus filled silencers with $\rho_f = 100$ g/L and 200 g/L densities. In comparison with Fig. 2 ($\phi = 8\%$), lower porosity ($\phi = 2\%$) shows a somewhat similar behavior, but (1) lowers the acoustic attenuation of higher density filling, and (2) in contrast to Fig. 2, the higher density filling results in a lower peak attenuation. Thus the smaller duct porosity weakens the acoustic performance of an absorbing silencer, particularly at higher filling densities. The impact of lower porosity on the deterioration of the acoustic attenuation with unfilled resonators at higher frequencies is, to a degree, reflected in filled resonators, as well.

Figure 7 compares the predictions with the experiment for a silencer with $\phi = 2\%$ and $\rho_f = 100$ g/L. Note that the one-dimensional predictions are now accurate in the frequency range of interest. For the predictions presented in Figs. 4, 5, and 7, it is important to note that Sullivan's empirical expression, Eq. (19), has been modified as Eq. (20) and used to account for the effect of absorbing material on the perforation impedance. Figure 8 compares the predictions based on Eqs. (19) and (20) for the silencer with $\phi = 2\%$ and $\rho_f = 200$ g/L. Using Eq. (19), both methods overestimate and underestimate the transmission loss below and above around 1500 Hz, respectively. In contrast, the predictions from the boundary element method with Eq. (20) show a good agreement with the experiment, and the predictions from the one-dimensional analysis are now closer to the experiment, while still overestimating the transmission loss around the peak frequency. Figure 8 clearly demonstrates that, for reasonable predictions, the effect of absorbing material on the perforation impedance needs to be accounted for.

4. CONCLUDING REMARKS

The acoustic performance of a single-pass perforated absorbing silencer is investigated experimentally and theoretically. The absorbing material enhances the noise reduction of the perforated duct silencer, while exhibiting a resonance-like attenuation behavior. The peak magnitude and frequency of the transmission loss depend on the duct porosity and absorbing material density. For $\phi = 8\%$, the silencer with $\rho_f = 200$ g/L shows higher transmission loss at lower frequency than that with $\rho_f = 100$ g/L. In contrast, for $\phi = 2\%$, the silencer with $\rho_f = 100$ g/L has better acoustic performance than that with $\rho_f = 200$ g/L. For silencers with $\phi = 8\%$, the boundary element method shows good agreement for the entire frequency range of interest, while the one-dimensional method is accurate only at low frequencies. For silencers with $\phi = 2\%$, both one-dimensional and boundary element methods compare reasonably well with the experiments. The study also illustrates the importance of the effect of absorbing material on the perforation impedance for accurate predictions.

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REFERENCES


NOMENCLATURE

\( c_0 \) speed of sound in air
\( \bar{c} \) complex speed of sound in the absorbing material
\( d_1 \) perforated duct inner diameter
\( d_2 \) outer chamber inner diameter
\( d_h \) perforate hole diameter
\( f \) frequency
\( i \) imaginary unit \((= \sqrt{-1})\)
\( k \) wave number in air
\( k \) complex wave number in the absorbing material
\( l \) silencer length
\( p_1 \) acoustic pressure in the perforated duct
\( p_2 \) acoustic pressure in the outer chamber
\( R \) flow resistivity of the absorbing material
\( T_i \) transfer matrix elements
\( w_t \) wall thickness of the perforated duct
\( T_L \) transmission loss
\( u_1 \) acoustic velocity in the perforated duct
\( u_2 \) acoustic velocity in the outer chamber
\( x \) co-ordinate axis

Greek Symbols
\( \phi \) perforated duct porosity
\( \rho_f \) absorbing fiber material bulk density
\( \rho_0 \) air density
\( \bar{\rho} \) complex dynamic density in the absorbing material
\( \zeta_p \) acoustic impedance of perforate

Subscripts
\( 0 \) air
\( 1 \) perforated duct
\( 2 \) outer chamber
\( f \) absorbing fiber material
\( h \) perforate hole
\( p \) perforate
\( w \) perforated duct wall

Superscripts
\( i \) inlet
\( o \) outlet
\( p \) perforate