Analytical approach for sound attenuation in perforated dissipative silencers

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A two-dimensional analytical solution is developed to determine the acoustic performance of a perforated single-pass, concentric cylindrical silencer filled with fibrous material. To account for the wave propagation through absorbing fiber and perforations, the complex characteristic impedance, wave number, and perforation impedance are employed. With expressions for the eigenvalues and eigenfunctions of sound propagation in the perforated dissipative chamber, the transmission loss is obtained by applying a pressure and velocity matching technique. The results from the analytical method are then compared with both experiments and numerical predictions based on the boundary element method (BEM), showing a reasonable agreement. The effects of geometry, fiber properties, and perforation porosity on the acoustic attenuation performance are discussed in detail. ©2004 Acoustical Society of America. [DOI: 10.1121/1.1694994]

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I. INTRODUCTION

Dissipative silencers are used to control noise in automotive exhaust systems due to their broadband attenuation characteristics (particularly at mid to high frequencies) and low back pressure. Thus far, a number of numerical techniques have been used to quantify their acoustic behavior, including the finite element and boundary element methods. Given the computational effort for sufficiently accurate numerical results, however, an analytical approach may be a viable alternative for silencers with simple cross sections, such as circular. In terms of bulk-reacting model, the acoustic attenuation in infinite rectangular and circular lined ducts was analyzed by Scott; the characteristics of sound transmission in infinite rectangle, annular, and circular lined ducts with mean flow were investigated by Ko; the effects of a perforated screen and mean flow on the modal attenuation rates in infinite circular lined ducts were examined by Nilsson and Brander, and Cummings and Chang, respectively. Cummings and Chang further investigated a finite-length dissipative silencer including mean flow and obtained the transmission loss by a mode-matching technique using the continuity of acoustic pressure and axial particle velocity across the silencer discontinuities. Peat obtained the transfer matrix for a dissipative expansion chamber by a “low-frequency approximation,” which is accurate at low frequencies. For a circular dissipative expansion chamber with the fibrous material separated from the central airway by a perforated screen, Kirby obtained the transmission loss using a technique similar to the low-frequency approximation by including additional terms in the series expansions for the Bessel and Neumann functions. This approach, however, remains confined to relatively low frequencies due to the inherent limitation of expansions with a finite number of terms. Wang developed a one-dimensional (1D) decoupling method to study the acoustic attenuation of a resonator with fiber in the expansion chamber. Selamet et al. investigated the sound attenuation in single-pass concentric perforated dissipative silencers by a 1D analytical approach, BEM, and experiments. Munjal and Thawani examined the effect of a perforated protective plate or a thin impervious layer on the acoustic performance of lined circular ducts and parallel-baffle mufflers. For both cases, they presented analytical models with bulk-reacting as well as locally reacting absorptive linings, while accounting for the effect of grazing flow on the impedance of perforated plate. Munjal also investigated a pod silencer consisting of a lined circular duct with a cylindrical pod inside, and obtained the four-pole parameters and transmission loss. By a pressure and velocity matching technique originated in Ref. 17, a closed-form, two-dimensional analytical solution was developed to investigate the sound attenuation in a circular dissipative expansion chamber.

In Ref. 18, no perforated screen was considered in the dissipative chamber, leaving fiber in direct contact with the gas in the central airway. However, to retain the fibrous material, a perforated screen is usually placed between the fiber and the central airway, which impacts the acoustic performance of the silencer. The acoustic impedance of a porous layer with perforated facing was investigated by Ingard and Bolt. Kirby and Cummings obtained the acoustic impedance of perforated plates by examining experimentally two types of perforations, circular and louvered plates, with and without porous backing. Empirical formulas and semiempirical predictions were provided for the case without, and with fiber backing layer, respectively. The objective of the present study is to extend an earlier work by introducing the perforated screen into a single-pass, concentric, dissipative cy-
lindrical silencer. Following this introduction, Sec. II obtains the eigenvalues and eigenfunctions of the acoustic wave in the perforated absorbing chamber, and develops a two-dimensional analytical approach for the transmission loss by applying the pressure/velocity matching technique. Section III compares the analytical predictions with numerical results and experimental measurements, and discusses the effect of geometry, fiber resistance, and porosity on the transmission loss. The study is concluded with final remarks in Sec. IV.

II. ANALYTICAL APPROACH

Consider a cylindrical chamber of length \( L \) and radius \( r_3 \), with sound-absorbing material placed between radii \( r_2 \) and \( r_1 \), as shown in Fig. 1. The inlet and outlet pipes of radius \( r_1 \) are designated as domains I and III, and the chamber as II_a and II_b. A perforated screen with porosity \( \phi \) is located at \( r = r_2 \) to separate the central airway (domain II_a) from the absorbing material (domain II_b) in the expansion chamber. The absorbing material is assumed to be homogeneous and isotropic, characterized by the complex speed of sound \( \tilde{c} \) and density \( \tilde{\rho} \) (The expressions are deferred to Appendix A).

A. Wave propagation in the inlet pipe (domain I)

For a two-dimensional axisymmetric wave propagation in the inlet duct (domain I), the solution of the Helmholtz equation is expressed as

\[
P_A(r,x) = \sum_{n=0}^{\infty} \left( A_n^+ e^{-jk_{r,A,n}r} + A_n^- e^{jk_{r,A,n}r} \right) \psi_{A,n}(r),
\]

with \( j = \sqrt{-1} \), \( P_A \) being the acoustic pressure in the inlet pipe; \( A_n^+ \) and \( A_n^- \) the modal amplitudes corresponding to components traveling in the positive and negative \( x \) directions, respectively; \( k_{r,A,n} \) the wave number in \( r \) direction with subscript \( x, A, n \) denoting axial direction, domain I, and order of the waves, respectively;

\[
\psi_{A,n}(r) = J_0(k_{r,A,n}r),
\]

(2)

is the eigenfunction for this circular duct with subscript \( r \) designating the radial direction and \( J_0 \) being the zeroth-order Bessel function of the first kind, \( k_{r,A,n} \) the radial wave number satisfying the rigid wall boundary condition of

\[
J_0'(k_{r,A,n}r_1) = 0.
\]

(3)

The axial wave number of the \( n \) mode is

\[
k_{x,A,n} = \left\{ \begin{array}{ll}
\sqrt{k_0^2 - k_{r,A,n}^2}, & k_0 > k_{r,A,n}, \\
-j\sqrt{k_{r,A,n}^2 - k_0^2}, & k_0 < k_{r,A,n},
\end{array} \right.
\]

(4)

where \( k_0 = 2\pi f/c_0 \) is the wave number in air, \( f \) the frequency, and \( c_0 \) the speed of sound. In view of the linearized momentum equation, the particle velocity in the axial direction may then be written as

\[
u_{x,A}(r,\tilde{\theta}) = \frac{1}{\tilde{\rho}_0 \omega} \sum_{n=0}^{\infty} k_{x,A,n}
\times (A_n^+ e^{-jk_{x,A,n}\tilde{\theta}} - A_n^- e^{jk_{x,A,n}\tilde{\theta}}) \psi_{A,n}(r),
\]

(5)

where \( \tilde{\rho}_0 \) is the density of air and \( \omega \) the angular velocity.

B. Wave propagation in the expansion chamber (domains II_a and II_b)

For the expansion chamber, the two-dimensional axisymmetric wave propagation in cylindrical coordinates \((r,x)\) is governed by

\[
\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial x^2} + \kappa^2 P = 0,
\]

(6)

where

\[
\kappa = \begin{cases} 
\tilde{k}, & 0 \leq r \leq r_2, \\
\tilde{k} \tilde{r}, & r_2 \leq r \leq r_3,
\end{cases}
\]

(7)

with \( \tilde{k} = 2\pi f/\tilde{c} \) denoting the wave number in the fibrous material.

The central airway and fiber material have the same axial wave number \( k_{x,B,n} \) and different radial wave numbers \( k_{r,B,n} \) (for the air) and \( \tilde{k}_{r,B,n} \) (for the fiber) related by

\[
k_{r,B,n}^2 + k_{x,B,n}^2 = k_0^2,
\]

(8)

and

\[
\tilde{k}_{r,B,n}^2 + k_{x,B,n}^2 = \tilde{k}^2.
\]

(9)

The acoustic pressure is expressed as

\[
P_B(r,x) = \sum_{n=0}^{\infty} \left( B_n^+ e^{-jk_{x,B,n}\tilde{\theta}} + B_n^- e^{jk_{x,B,n}\tilde{\theta}} \right) \psi_{B,n,p}(r),
\]

(10)

where

\[
P_B(r,x) = \begin{cases} 
P_{II_a}(r,x), & 0 \leq r \leq r_2, \\
P_{II_b}(r,x), & r_2 \leq r \leq r_3,
\end{cases}
\]

(11a)
where \( \rho_0 \) denotes the zeroth-order Neumann function; \( C_1, C_2, C_3, \) and \( C_4 \) the coefficients related by the following four boundary conditions at \( r = 0, r_2, \) and \( r_3 \):

1. At \( r = 0 \), the pressure is finite; thus, Eqs. (10), (11), and (14) yield
   \[ C_2 = 0. \] (15)

2. From Eqs. (10), (11), and (14), the rigid wall boundary condition at \( r = r_3 \) gives
   \[ u_{r,B} = \frac{j}{\rho_0} \frac{\partial P_B}{\partial r}, \] (16a)
   \[ C_3 = \frac{k_{r,B,n}}{k_{r,B,n} \rho_0} \frac{J_1(k_{r,B,n} r_2)}{J_1(k_{r,B,n} r_3)} Y_1(k_{r,B,n} r_3) J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3) - J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3) J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3), \] (17a)
   \[ C_4 = \frac{J_1(k_{r,B,n} r_3)}{Y_1(k_{r,B,n} r_3) C_3 = - \frac{k_{r,B,n} \rho_0}{k_{r,B,n} \rho_0} J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3) J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3) - J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3) J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3), \] (17b)

and the characteristic equation
\[
\frac{\rho_0 \tilde{r}_{r,n}}{k_{r,B,n}} \left[ J_0(k_{r,B,n} r_2) \right] J_1(k_{r,B,n} r_2) + \frac{\partial P_B}{\partial r} \left[ J_0(k_{r,B,n} r_2) \right] J_1(k_{r,B,n} r_2) = \left[ J_0(k_{r,B,n} r_2) \right] \frac{J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3) - J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3)}{Y_1(k_{r,B,n} r_3) \frac{J_1(k_{r,B,n} r_2)}{J_1(k_{r,B,n} r_2) - Y_0(k_{r,B,n} r_2) J_1(k_{r,B,n} r_2)} \frac{Y_1(k_{r,B,n} r_3) - Y_1(k_{r,B,n} r_2) J_1(k_{r,B,n} r_3) - Y_1(k_{r,B,n} r_2) J_1(k_{r,B,n} r_3), \] (18)

In view of Eqs. (8) and (9), Eq. (22) yields the solutions for the axial wave number \( k_{x,B,n} \) for a given frequency. (The details of the solution technique are deferred to Appendix C.)

The transverse eigenfunctions for the pressure can then be determined as
\[
\psi_{B,n}(r) = \begin{cases} J_0(k_{r,B,n} r), & 0 \leq r \leq r_2, \\ 0, & r_2 < r \leq r_3, \end{cases} \] (19)

with \( u_{r,B} \) being the radial particle velocity; \( J_1 \) and \( Y_1 \) the first-order Bessel and Neumann functions, respectively.

(3) From Eqs. (10), (11), and (14), the continuity of radial particle velocity \( u_r \) at \( r = r_2 \) yields
\[
u_{r,H_a} = \frac{\partial P_B}{\partial r} \left[ J_0(k_{r,B,n} r_2) \right] \frac{J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3) - J_1(k_{r,B,n} r_2) Y_1(k_{r,B,n} r_3)}{Y_1(k_{r,B,n} r_3) \frac{J_1(k_{r,B,n} r_2)}{J_1(k_{r,B,n} r_2) - Y_0(k_{r,B,n} r_2) J_1(k_{r,B,n} r_2)} \frac{Y_1(k_{r,B,n} r_3) - Y_1(k_{r,B,n} r_2) J_1(k_{r,B,n} r_3) - Y_1(k_{r,B,n} r_2) J_1(k_{r,B,n} r_3), \] (20)

with \( \xi_n \) (given in Appendix B) being the nondimensionalized acoustic impedance of the perforated screen.

By assuming
\[ C_1 = 1, \] (21)

Eqs. (15)–(18) yield
\[ \psi_{B,n}(r) = \begin{cases} J_0(k_{r,B,n} r), & 0 \leq r \leq r_2, \\ 0, & r_2 < r \leq r_3, \end{cases} \] (22)

with
\[ u_{r,B} = 0, \] for \( r = r_3, \] (23)
Equations (10), (11), and (23) yield
\[
P_B(r) = \begin{cases} 
\sum_{n=0}^{\infty} (B_n^+ e^{-j k_n r} + B_n^- e^{j k_n r}) J_0(k_n r), & 0 \leq r \leq r_1, \\
\sum_{n=0}^{\infty} (B_n^+ e^{-j k_n r} + B_n^- e^{j k_n r}) C_5 \left[ J_0(k_n r) - \frac{J_1(k_n r)}{Y_1(k_n r)} \right], & r_2 \leq r \leq r_3.
\end{cases}
\]

From the linearized momentum equation
\[
\begin{align*}
&u_x(r) = \frac{j}{\rho_0 \omega} \frac{\partial P_H}{\partial x}, & 0 \leq r \leq r_2, \\
&\frac{j}{\rho \omega} \frac{\partial P_H}{\partial x}, & r_2 \leq r \leq r_3,
\end{align*}
\]
the particle velocity in the axial direction is then obtained as
\[
u_x(r) = \begin{cases} 
J_0(k_n r), & 0 \leq r \leq r_1, \\
C_5 \frac{\rho_0}{\rho} \left[ J_0(k_n r) - \frac{J_1(k_n r)}{Y_1(k_n r)} \right], & r_2 \leq r \leq r_3,
\end{cases}
\]
with
\[
\psi_{B,n,A}(r) = \begin{cases} 
J_0(k_n r), & 0 \leq r \leq r_1, \\
C_5 \frac{\rho_0}{\rho} \left[ J_0(k_n r) - \frac{J_1(k_n r)}{Y_1(k_n r)} \right], & r_2 \leq r \leq r_3,
\end{cases}
\]
being the transverse modal eigenfunction for the velocity.

C. Wave propagation in the outlet pipe (domain III)

The acoustic pressure and axial velocity in the outlet pipe (domain III) are similar to those in the inlet pipe (domain I) and expressed as
\[
P_C(r, x) = \sum_{n=0}^{\infty} (C_n^+ e^{-j k_n x} \psi_{C,n}(x-L) + C_n^- e^{j k_n x} \psi_{C,n}(x-L))
\]
and
\[
u_C(r, x) = \frac{1}{\rho_0 \omega} \sum_{n=0}^{\infty} k_n \psi_{C,n}(C_n^+ e^{-j k_n x} \psi_{C,n}(x-L) - C_n^- e^{j k_n x} \psi_{C,n}(x-L)),
\]
where the subscript C denotes domain III, $C_n^+$ and $C_n^-$ are the amplitudes; with both the eigenfunction $\psi_{C,n}(r)$ and wave number $k_n$ being similar to those in the inlet pipe.

D. Transmission loss prediction

With the expressions of pressure and particle velocity of the inlet, outlet, and expansion chamber [Eqs. (1), (5), and (24–26)], transmission loss can then be obtained by solving the unknown coefficients $A_n$, $B_n$, and $C_n$ using the boundary conditions at the expansion ($x = 0$) and contraction ($x = L$). At the interfaces of the expansion and contraction, the acoustic pressure and velocity continuity conditions reveal
\[
P_A = P_B, \quad 0 \leq r \leq r_1, \quad x = 0, \tag{27a}
\]
\[
u_A = \begin{cases} 
u_x, & 0 \leq r \leq r_1, \quad x = 0, \\
0, & r_1 \leq r \leq L, \quad x = 0, \tag{27b}
\end{cases}
\]
\[
P_C = P_B, \quad 0 \leq r \leq r_1, \quad x = L, \tag{27c}
\]
\[
u_C = \begin{cases} 
u_x, & 0 \leq r \leq r_1, \quad x = L, \\
0, & r_1 \leq r \leq L, \quad x = L. \tag{27d}
\end{cases}
\]
In view of the expressions of the pressure and velocity as infinite series of unknown amplitudes in Eqs. (1), (5), and (24–26), Eq. (27) gives
\[
\sum_{n=0}^{\infty} (A_n^+ + A_n^-) \psi_{A,n}(r) = \sum_{n=0}^{\infty} (B_n^+ + B_n^-) \psi_{B,n,p}(r), \quad 0 \leq r \leq r_1, \tag{28a}
\]
\[
\sum_{n=0}^{\infty} k_{x,B,n}(B_n^+ - B_n^-) \psi_{B,n,u}(r) = \sum_{n=0}^{\infty} k_{x,A,n}(A_n^+ - A_n^-) \psi_{A,n}(r), \quad 0 \leq r \leq r_1, \tag{28b}
\]
\[
\sum_{n=0}^{\infty} (C_n^+ + C_n^-) \psi_{C,n}(r) = \sum_{n=0}^{\infty} (B_n^+ e^{-j k_n x} + B_n^- e^{j k_n x}) \psi_{B,n,p}(r), \quad 0 \leq r \leq r_1, \tag{28c}
\]
\[ \sum_{n=0}^{\infty} k_{x,B,n}(B_n^+ e^{-jk_{x,B,n}L} - B_n^- e^{jk_{x,B,n}L}) \psi_{B,n,u_1}(r) \]

\[ = \sum_{n=0}^{\infty} k_{x,C,n}(C_n^+ - C_n^-) \psi_{C,n}(r), \quad \text{for } 0 \leq r \leq r_1, \]

\[ = 0, \quad \text{for } r_1 \leq r \leq r_3. \]

In Eq. (28), the infinite series of unknown amplitudes need to be truncated to a suitable number. Then, the same number of equations is solved for the amplitudes of acoustic waves. An approach proposed in Ref. 18 is adopted here to match the sound field. Imposing the continuity of the integral of the pressure and axial velocity over discrete zones of the interfaces at the expansion (\(x=0\)) and contraction (\(x=L\)), Eq. (28) yields the pressure and velocity matching conditions as

\[ \sum_{n=0}^{N} (A_n^+ + A_n^-) \int_{r_m,p}^{r_m,u} \psi_{A,n}(r) dr \]

\[ = \sum_{n=0}^{N} (B_n^+ + B_n^-) \int_{r_m,p}^{r_m,u} \psi_{B,n,p}(r) dr, \quad (29a) \]

\[ \sum_{n=0}^{N} k_{x,B,n}(B_n^+ - B_n^-) \int_{r_m,u}^{r_m,a} \psi_{B,n,u_u}(r) dr \]

\[ = \left\{ \begin{array}{l}
\sum_{n=0}^{N} k_{x,A,n}(A_n^+ - A_n^-) \int_{r_m,a}^{r_m,u} \psi_{A,n}(r) dr, \\
\quad \text{for } 0 \leq r_m,a \leq r_1, \\
\sum_{n=0}^{N} k_{x,A,n}(A_n^+ - A_n^-) \int_{r_1}^{r_m,u} \psi_{A,n}(r) dr, \\
\quad \text{for } r_1 \leq r_m,a \leq r_3, \\
\sum_{n=0}^{N} (C_n^+ + C_n^-) \int_{r_m,p}^{r_m,a} \psi_{C,a}(r) dr \\
\quad \text{for } 0 \leq r_m,a \leq r_1, \\
\sum_{n=0}^{N} k_{x,C,n}(C_n^+ - C_n^-) \int_{r_1}^{r_m,a} \psi_{C,a}(r) dr, \\
\quad \text{for } r_1 \leq r_m,a \leq r_3, \\
\sum_{n=0}^{N} k_{x,C,n}(C_n^+ - C_n^-) \int_{r_1}^{r_m,a} \psi_{C,a}(r) dr, \\
\quad \text{for } r_1 \leq r_m,a \leq r_3, \\
\end{array} \right. \quad (29b) \]

\[ \int_{r_m,p}^{r_m,u} \psi_{B,n,p}(r) dr, \quad (29c) \]

\[ \int_{r_m,u}^{r_m,a} \psi_{B,n,u}(r) dr \]

\[ = \left\{ \begin{array}{l}
\sum_{n=0}^{N} k_{x,C,n}(C_n^+ - C_n^-) \int_{r_m,a}^{r_m,u} \psi_{C,a}(r) dr, \\
\quad \text{for } 0 \leq r_m,a \leq r_1, \\
\sum_{n=0}^{N} k_{x,C,n}(C_n^+ - C_n^-) \int_{r_1}^{r_m,u} \psi_{C,a}(r) dr, \\
\quad \text{for } r_1 \leq r_m,a \leq r_3, \\
\end{array} \right. \quad (29d) \]

with

\[ r_{m,p} = \frac{m}{N+1} r_1, \quad m = 1, \ldots, N + 1; \quad (30a) \]

In view of 4(N+1) coefficients solved in Eq. (29) and the assumptions that: (1) the incoming wave being planar and \(A_0^+\) be unity; (2) an anechoic termination be imposed at the exit by setting \(C_n^-\) to zero; and (3) all transmitted waves in the outlet pipe are nonpropagating modes except the first mode with \(C_0^-\), the transmission loss is then determined as

\[ TL = -20 \log_{10} |C_0^+|. \quad (31) \]

### III. RESULTS AND DISCUSSION

A single-pass concentric silencer with acoustic absorbing material and uniformly perforated duct is studied here (recall Fig. 1). Parameters of the basic model are given as \(L = 25.72\) cm, \(r_1 = 2.45\) cm, \(r_2 = 2.45\) cm, \(r_3 = 8.22\) cm, \(R = 4896\) Rayls/m (for a filling density of 100 g/l), duct porosity \(\phi = 8\%\), thickness of the perforated duct \(t_u = 0.09\) cm, and diameter of the holes \(d_h = 0.249\) cm. The two-microphone technique and BEM are also used to obtain the transmission loss experimentally and computationally.

To check the accuracy of analytical approach for the basic model, Fig. 2 presents the transmission loss determined from the analytical approach (with different numbers of higher order modes), 3D BEM, and the experiments. For the current geometry and frequency range of interest, the analytical results are accurate enough for \(N \geq 9\), which are essentially identical to those for \(N = 12\) to the degree that the latter cannot be distinguished in the figure. These results agree well with the BEM predictions and show a reasonable comparison with the measurements.

Analytical results for absorbing silencers with different flow resistivities (\(R = 1000, 4896, \) and 17378 Rayls/m) are presented in Fig. 3, in contrast to an empty chamber. Higher flow resistivity tends to improve the performance of the silencer at medium to high frequencies, while moving the transmission loss peak to lower frequencies. At low frequencies, the transmission loss is reduced with flow resistivity. The acoustic performance of the silencer with \(R = 4896\) Rayls/m is better than the one with no absorbing material nearly at all frequencies, however, the transmission loss of the silencer with \(R = 17378\) Rayls/m deteriorates slightly below 250 Hz.

Analytical results for silencers with different perforation porosities (\(\phi = 2\%, \ 8\%\), and 50%) are presented in Fig. 4, including the limiting case of removed screen. At relatively high frequencies, the attenuation is significantly improved with porosity, while shifting the peak to higher frequencies. Higher perforations, however, tend to yield somewhat lower TL at low frequencies. The silencer with \(\phi = 2\%\) exhibits a behavior similar to that of a Helmholtz resonator, with its resonance frequency around 700 Hz and a rapidly decreasing transmission loss with frequency. The performance of the silencer with \(\phi = 50\%\) is close to the one without the perforated screen, as expected.

The analytical results with varying fiber thickness (\(r_2 = 2.45, 3.45, \) and 4.45 cm) are depicted in Fig. 5. Similar to a dissipative expansion chamber without perforated screen, 18
increasing the fiber thickness improves the attenuation particularly at mid to high frequencies. The location of peak attenuation is, however, almost maintained.

Figure 6 shows the effect of outer chamber radius \( r_3 = 5.055, 8.22, \) and 10 cm. Increasing \( r_3 \) improves the performance of the absorbing silencers at low frequencies, while shifting the peak attenuation to lower frequencies. Unlike reactive chambers, however, the larger \( r_3 \) does not necessarily lead to higher attenuation peaks. Figure 7 illustrates the effect of chamber length \( L = 15, 25.72, \) and 40 cm) on the transmission loss of absorbing silencers. As expected, increasing \( L \) improves the attenuation at all frequencies.

IV. CONCLUSIONS

A two-dimensional, closed-form analytical approach has been developed to predict the acoustic attenuation of a perforated single-pass, concentric cylindrical expansion chamber filled with fibrous material. By using the boundary conditions at the central, perforated screen, and rigid wall, the governing eigenequation is obtained, which determines the sound field in the dissipative chamber. With the eigenvalues and eigenfunctions solved from the eigenequation, the transmission loss is predicted by applying the pressure and particle velocity matching at the interfaces of the expansion and contraction. The analytical predictions show a reasonable agreement with the experimental and BEM results for the frequency range of interest. The present study also illustrates the effects of chamber geometry, fiber parameters, and porosity on the acoustic attenuation of perforated dissipative silencers.

APPENDIX A: ACOUSTICAL PROPERTIES OF FIBER MATERIAL

The absorption of acoustic waves in fibrous material is mainly due to viscous dissipation, which involves complex-
valued characteristic impedance \( \tilde{Z} = \tilde{\rho} \tilde{c} \) and wave number \( \tilde{k} = 2 \pi f / \tilde{c} \). For the absorbing material in the present study, \( \tilde{Z} \) and \( \tilde{k} \) are given as:

\[
\frac{\tilde{Z}}{Z_0} = \left[ 1 + 0.0855(f/R)^{-0.754} \right] + j \left[ -0.0765(f/R)^{-0.732} \right],
\]

(A1)

\[
\frac{\tilde{k}}{k} = \left[ 1 + 0.1472(f/R)^{-0.577} \right] + j \left[ -0.1734(f/R)^{-0.595} \right],
\]

(A2)

where \( Z_0 = \rho_0 c_0 \) is the characteristic impedance of the air; \( R \) [mks Rayls/m] denotes the flow resistivity. The measured flow resistivities \( R \) are 4896 and 17 378 Rayls/m, corresponding to material densities of 100 and 200 g/l, respectively.

APPENDIX B: ACOUSTIC IMPEDANCE OF PERFORATES

In Eq. (18), the nondimensionalized perforate acoustic impedance \( \tilde{\xi}_p \) relates the acoustic pressure in the inner duct and outer chamber through the interface. Sullivan and Crocker\(^{22} \) presented an empirical expression for the acoustic impedance of perforate holes as:

\[
\tilde{\xi}_p = \left[ 0.006 + j k_0 (t_w + 0.75 d_h) \right] / \phi.
\]

(B1)

In the presence of fiber (for perforations facing absorbing material), Eq. (B1) has been modified by Selamet et al.\(^{4} \) as:

\[
\tilde{\xi}_p = \left[ 0.006 + j k_0 \left( t_w + 0.375 d_h \left( 1 + \frac{\tilde{Z}}{Z_0} \frac{\tilde{k}}{k_0} \right) \right) \right] / \phi,
\]

(B2)

which is also used in this study.
APPENDIX C: SOLUTION OF THE CHARACTERISTIC EQUATION

The axial wave numbers are obtained from the solution of

\[ F(\kappa, \tilde{\kappa}, \tilde{\kappa}) = 0, \]  

where

\[ k_x^2 + k_z^2 = k_0^2, \quad \tilde{k}_x^2 + \tilde{k}_z^2 = \tilde{k}_0^2. \]

Given an initial approximation for \( k_x,0 \) in the secant method used here, the root of Eq. (C1) is obtained by the loop

\[ k_{x,i+1} = k_{x,i} - \frac{F(R, \tilde{\kappa}, k_x, f)}{F(R, \tilde{\kappa}, (1+\Delta)k_x, f) - F(R, \tilde{\kappa}, (1-\Delta)k_x, f)} (1+\Delta)k_{x,i} - (1-\Delta)k_{x,i} \]

where \( i = 0, 1, 2, ..., i_{\text{max}} \), \( \Delta \) is a small constant, and \( i_{\text{max}} \) is an integer designating the loop number. The selection of a suitable initial guess for the desired axial wave number \( k_x,0 \) is critical in this approach. For example, Cummings\cite{10,11} obtained the roots of a dissipative silencer by the secant method with initial values chosen from a rectangular grid in the complex plane, which may cause “jumping” of eigenvalues. The present work avoids such potential eigenvalue jumping by determining the initial values based on a method.
originated from the solution of dispersion equations for shell–fluid coupled systems \(^{23-27}\).

Since the characteristic impedance and wave number in fibrous materials approach the corresponding values in air at very high frequencies, the roots of Eq. (C1) are obtained first at the upper-frequency limit of the range of interest, \(f_{\text{max}}\). Let \(R = 0\) and \(\tilde{z}_p = 0\) in \(F(R, \tilde{z}_p, k_x, f_{\text{max}})\) of Eq. (C1). The solution of \(F(0,0, k_x, f_{\text{max}})=0\) then yields the axial wave numbers of a duct without fiber or perforation at \(f_{\text{max}}\): 
\[k_1(0,0, f_{\text{max}}), k_2(0,0, f_{\text{max}}), \ldots\]
with the superscripts denoting the mode order. With the foregoing initial approximation, the roots of the characteristic equation
\[
F \left( \frac{R}{N_{R, \tilde{z}_p}}, \frac{\tilde{z}_p}{N_{R, \tilde{z}_p}} k_x, f_{\text{max}} \right) = 0
\]  
(C3)

(with \(N_{R, \tilde{z}_p}\) being a large integer), can be readily obtained from the secant method as 
\[k_1((R/N_{R, \tilde{z}_p}), (\tilde{z}_p/N_{R, \tilde{z}_p}), f_{\text{max}}), k_2((R/N_{R, \tilde{z}_p}), (\tilde{z}_p/N_{R, \tilde{z}_p}), f_{\text{max}}), \ldots\]
Since the axial wave numbers of Eq. (C3) are very close to those of 
\(F(0,0, k_x, f_{\text{max}})=0\), the secant method with such initial values is proved to be successful and avoids eigenvalue jumping. For \(n_{R, \tilde{z}_p} = 1,2, \ldots, N_{R, \tilde{z}_p}-1\), the roots of 
\[F((n_{R, \tilde{z}_p}+1)/N_{R, \tilde{z}_p})R, ((n_{R, \tilde{z}_p}+1)/N_{R, \tilde{z}_p})\tilde{z}_p, k_x, f_{\text{max}})=0\] are determined with the initial values estimated as the solutions of 
\[F((n_{R, \tilde{z}_p}+1)/N_{R, \tilde{z}_p})R, ((n_{R, \tilde{z}_p}+1)/N_{R, \tilde{z}_p})\tilde{z}_p, k_x, f_{\text{max}})=0\]. Thus, for the dissipative expansion chamber with flow resistivity \(R\) and perforation \(\tilde{z}_p\), the roots of the characteristic equation 
\(F(R, \tilde{z}_p, k_x, f_{\text{max}})=0\) have been obtained at the highest frequency of interest \(f_{\text{max}}\). With the results now available at \(f_{\text{max}}\), the axial wave numbers can then be determined at all frequencies by a similar secant method. The frequency is reduced by a suitable step \(\Delta f = 10\) Hz, for example, and the solution for the previous frequency is used as the initial values at the new frequency \(f_{\text{max}}-\Delta f\). The characteristic equation is thus solved at all frequencies, leading to the dispersion curves for the dissipative expansion silencer. This method is commonly adopted to get the dispersion curves for cylindrical shell–fluid coupled systems (both fluid-filled shell\(^{23-27}\)) and the shell immersed in fluid\(^{26}\). Numerical study demonstrates that this method avoids the eigenvalue jumping addressed in Refs. 10 and 11. In addition, the present approach can save significant computational time as compared to the numerical methods (BEM/FEM). For the basic model with 
\(r_1 = 2.45\) cm, \(r_2 = 2.45\) cm, \(r_3 = 8.22\) cm, \(R = 4896\) Rayls/m, and \(\phi = 8\%\), the computational time for the present analytical method is much less than that of BEM.