Dissipative expansion chambers with two concentric layers of fibrous material

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Abstract: The acoustic performance of a dissipative expansion chamber lined with two concentric, annular layers of fibrous material with different resistances is investigated. A two-dimensional analytical approach is used to determine the transmission loss of this dissipative silencer. From the boundary conditions at the rigid wall, and the interfaces between the fibre layers and the central airway, the characteristic function and thus eigenvalues and eigenfunctions for sound propagation in the dissipative chamber are obtained, leading to transmission loss through application of pressure and velocity matching. The effects of geometry and fibre properties on the acoustic attenuation are also discussed.

Keywords: acoustic performance; dissipative expansion chamber; muffler; sound absorbing material.


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1 Introduction

Dissipative silencers, generally in the form of expansion chambers lined with fibrous materials, are commonly used to control noise in ventilation and automotive exhaust systems. Numerical techniques have been used often to study their acoustic performance, including a number of 2-D approaches (Cummings, 1995; Glav, 1996, 2000) for axially uniform and arbitrary cross-sections, and 3-D approaches (boundary element (Seybert et al., 1998; Selamet et al., 2001) and finite element methods (Peat and Rathi, 1995; Astley and Cummings, 1987; SYSNOISE, 2001) for irregular shapes. A variety of analytical approaches has also been employed for dissipative silencers with special shapes (cylindrical chamber, in particular). Using a mode-matching technique, Cummings and Chang (1988) investigated the transmission loss of a dissipative cylindrical expansion chamber including internal mean flow in the fibre. By a model based on the approximation of the radial pressure profile, Auregan et al. (2001) investigated a bulk-reacting dissipative cylindrical silencer. By a ‘low-frequency approximation’ in the solution of the dissipation equation, Peat (1991) obtained the axial wave number, the transfer matrix, and the transmission loss in a dissipative expansion chamber. By employing pressure and axial velocity matching at the discontinuities, Xu et al. (2004) developed an analytical solution to investigate the sound attenuation in a circular dissipative expansion silencer; Selamet et al. (2004) further studied the effect of perforated ducts on the sound attenuation in a dissipative silencer.
Configurations beyond simple dissipative expansion chambers have also been examined to improve the acoustic performance. Selamet et al. (2003) designed and analysed a dissipative-reactive (hybrid) silencer consisting of two dissipative expansion chambers combined with a reactive component (a Helmholtz resonator) between them. Their numerical investigation demonstrated that the reactive component was able to enhance the acoustic performance of this hybrid silencer at low frequencies. To improve the acoustic attenuation of dissipative silencers, Munjal (2003) investigated a pod silencer, which is a lined circular duct with a cylindrical pod inside.

Other similar concepts [splitter-silencers (Ko, 1975; Cummings and Sormaz, 1993), parallel-baffle mufflers (Mechel, 1990a, 1990b), and bar-silencers (Nilsson and Söderqvist, 1983; Cummings and Astley, 1996)] were also considered to improve the noise attenuation. Ko (1975) investigated theoretically the sound attenuation of porous splitters used in infinite lined flow ducts; the sound propagation was investigated analytically for circular, annular and rectangular ducts, however, no experimental or numerical results were presented. Cummings and Sormaz (1993) investigated the sound propagation and attenuation of dissipative splitter-silencers used in rectangular ducts. Using the ‘locally reacting’ model, Mechel (1990a, 1990b) developed an exact analysis for the sound transmission through ‘baffle-type silencers’, while also providing numerical results. He considered two types of ‘baffle-type silencers’: an extended silencer with an infinite lateral extension and an endless repetition of the baffle elements, and a rigid duct silencer with several baffle elements placed within. Numerical predictions for transmission loss were presented with different incident duct modes. The contributions to the total TL from propagation and reflections were discussed. Nilsson and Söderqvist (1983) claimed that a ‘bar-type silencer’ can improve the sound attenuation in a dissipative silencer by both decreasing distance between sound-absorbing surfaces and increasing the thickness of the fibre material. Using the finite element approach and measurements, Cummings and Astley (1996) investigated a ‘bar-silencer’, which consists of rectangular prisms of dissipative material placed in a rectangular lattice arrangement in a rigid-walled rectangular duct. The uniform mean flow in the air passage was included, and the finite element predictions showed reasonable agreement with the measured transmission loss.

As Xu et al. (2004) has illustrated, the flow resistivity of the fibre in the dissipative chamber greatly influences the acoustic performance. Generally, the increasing resistance of fibre in the dissipative chamber improves the sound attenuation in the mid to high frequency range, while deteriorating to a degree at low frequencies. Thus, to improve the sound attenuation performance at all frequencies, it is a paradox to design a dissipative expansion chamber filled completely with a unique fibre. The present study considers a layered dissipative silencer to investigate the potential trade-offs. Thus a single-pass, concentric, cylindrical expansion chamber lined with two concentric annular fibre layers of different fibre resistance is examined primarily by an analytical approach along with an illustrative comparison to predictions from the finite element method (FEM). Following the introduction, Section 2 develops the eigenvalue and eigenfunction of the acoustic wave in the layered dissipative circular expansion chamber, yielding the transmission loss by a two-dimensional analytical approach in terms of pressure/velocity matching technique. Section 3 discusses the effect of geometry, fibre parameters, and porosity on the transmission loss. Section 4 concludes the study with final remarks.
2 Analytical approach

Consider a cylindrical dissipative chamber of length \( L \) and radius \( r_4 \), with two kinds of sound-absorbing material placed between radii \( r_2 \) and \( r_3 \) (domain II_b), and \( r_3 \) and \( r_4 \) (domain II_c), respectively, as shown in Figure 1. The inlet and outlet ducts of radius \( r_1 \) are designated by domains I and III, and the chamber by II (domain II_a being the central airway, II_b and II_c being the fibre layers. This notation will be used hereafter as subscripts to distinguish domains.) The absorbing material is assumed to be homogeneous and isotropic, characterised by the complex speed of sound and density.

Figure 1  Geometry of a layered dissipative expansion chamber

2.1 Wave propagation for the dissipative chamber (domains II_a – II_c)

The expansion chamber of circular cross-section is filled with two fibre layers of thicknesses \( r_3 – r_2 \) (domain II_b) and \( r_4 – r_3 \) (domain II_c). Three domains (II_a – II_c) are characterised by the speed of sound \( c_{n_a}, \bar{c}_{n_b}, \bar{c}_{n_c} \), and density \( \rho_{n_a}, \bar{\rho}_{n_b}, \bar{\rho}_{n_c} \). The two-dimensional sound wave propagation in cylindrical coordinates \((r, x)\), is governed by

\[
\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial x^2} + \kappa^2 P = 0,
\]

where

\[
\kappa = \begin{cases} 
  k_{n_a}, & 0 \leq r \leq r_2 \\
  k_{n_b}, & r_2 \leq r \leq r_3 \\
  k_{n_c}, & r_3 \leq r \leq r_4 
\end{cases}
\]

with \( k_{n_a} = 2\pi f / c_{n_a} \), \( k_{n_b} = 2\pi f / \bar{c}_{n_b} \), and \( k_{n_c} = 2\pi f / \bar{c}_{n_c} \), denoting the wave numbers in domains II_a – II_c, and \( f \) being the frequency.
The central airway and the fibrous material have the common axial wave number \(k_{x,B,n}\) and different radial wave numbers \(k_{r,II,n}\) (for domain IIa), \(\tilde{k}_{r,II,n}\) (for domain IIb), and \(\tilde{k}_{r,II,n}\) (for domain IIc), which are related by

\[
\kappa_{r,B,n} = \begin{cases} 
\frac{k_{r,II,n}}{r}, & 0 \leq r \leq r_2 \\
\frac{k_{r,II,n}}{r_2}, & r_2 \leq r \leq r_3 \\
\frac{k_{r,II,n}}{r_3}, & r_3 \leq r \leq r_4 
\end{cases}
\]  

(3a)

\[
k_{x,B,n}^2 + k_{r,II,n}^2 = \kappa_{II,n}^2 
\]  

(3b)

\[
k_{x,B,n}^2 + k_{r,II,n}^2 = \tilde{k}_{II,n}^2 
\]  

(3c)

and

\[
k_{x,B,n}^2 + k_{r,II,n}^2 = \tilde{k}_{II,n}^2
\]  

(3d)

with the subscript \(x, B, n\) denoting axial direction, domain II, and order of the waves, respectively. The acoustic pressure, axial and radial particle velocities are then expressed as

\[
P(B, r, x) = \sum_{n=0}^{\infty} (B_n^+ e^{-i k_{x,B,n} x} + B_n^- e^{i k_{x,B,n} x}) \psi_{B,n,p}(r),
\]  

(4)

\[
u_{x,B}(r, x) = \frac{1}{\rho_{II, n} \omega} \sum_{n=0}^{\infty} \kappa_{x,B,n} (B_n^+ e^{-i k_{x,B,n} x} - B_n^- e^{i k_{x,B,n} x}) \psi_{B,n,u_x}(r),
\]  

(5)

and

\[
u_{r,B}(r, x) = \frac{j}{\rho_{II, n} \omega} \sum_{n=0}^{\infty} (B_n^+ e^{-i k_{x,B,n} x} + B_n^- e^{i k_{x,B,n} x}) \psi_{B,n,u_r}(r),
\]  

(6)

where \(\omega = 2\pi f\) is the angular velocity. Here

\[
P(B, r, x) = \begin{cases} 
P_{II, a}(r, x), & 0 \leq r \leq r_2 \\
P_{II, b}(r, x), & r_2 \leq r \leq r_3 \\
P_{II, c}(r, x), & r_3 \leq r \leq r_4
\end{cases}
\]  

(7a)

\[
u_{x,B}(r, x) = \begin{cases} 
u_{x,II_a}(r, x), & 0 \leq r \leq r_2 \\
u_{x,II_b}(r, x), & r_2 \leq r \leq r_3 \\
u_{x,II_c}(r, x), & r_3 \leq r \leq r_4
\end{cases}
\]  

(7b)

\[
u_{r,B}(r, x) = \begin{cases} 
u_{r,II_a}(r, x), & 0 \leq r \leq r_2 \\
u_{r,II_b}(r, x), & r_2 \leq r \leq r_3 \\
u_{r,II_c}(r, x), & r_3 \leq r \leq r_4
\end{cases}
\]  

(7c)
\[\psi_{\text{B},n,p}(r) = \begin{cases} \psi_{\text{IL},n,p}(r), & 0 \leq r \leq r_2 \\ \psi_{\text{IL},n,p}(r), & r_2 \leq r \leq r_3 \\ \psi_{\text{IL},n,p}(r), & r_3 \leq r \leq r_4 \end{cases} \]  
(7d)

\[\psi_{\text{B},n,ax}(r) = \begin{cases} \psi_{\text{IL},n,ax}(r) = \frac{\rho_{\text{ho}}}{\rho_{\text{hi}}} \psi_{\text{IL},n,p}(r), & 0 \leq r \leq r_2 \\ \psi_{\text{IL},n,ax}(r) = \frac{\rho_{\text{ho}}}{\rho_{\text{hi}}} \psi_{\text{IL},n,p}(r), & r_2 \leq r \leq r_3 \\ \psi_{\text{IL},n,ax}(r) = \frac{\rho_{\text{ho}}}{\rho_{\text{hi}}} \psi_{\text{IL},n,p}(r), & r_3 \leq r \leq r_4 \end{cases} \]  
(7e)

\[\psi_{\text{B},n,ax}(r) = \begin{cases} \frac{\partial \psi_{\text{IL},n,p}(r)}{\partial r}, & 0 \leq r \leq r_2 \\ \frac{\partial \psi_{\text{IL},n,p}(r)}{\partial r}, & r_2 \leq r \leq r_3 \\ \frac{\partial \psi_{\text{IL},n,p}(r)}{\partial r}, & r_3 \leq r \leq r_4 \end{cases} \]  
(7f)

where subscript \(r\) denotes the radial direction; \(B^+_n\) and \(B^-_n\) the modal amplitudes; \(\psi_{\text{B},n,p}, \psi_{\text{B},n,ax},\) and \(\psi_{\text{B},n,ax}\) the radial eigenfunctions for the pressure, axial and radial particle velocities, respectively. Substituting equations (4)–(7) into equation (1) gives

\[\frac{\partial^2 \psi_{\text{B},n,p}(r)}{\partial r^2} + \frac{1}{r} \frac{\partial \psi_{\text{B},n,p}(r)}{\partial r} + \kappa_{\text{r,B,ax}}^2 \psi_{\text{B},n,p}(r) = 0\]  
(8)

with the solution being expressed as

\[\psi_{\text{B},n,p}(r) = \begin{cases} C_1 J_0(\tilde{k}_{\text{r,IL},n}\ r) + C_2 Y_0(\tilde{k}_{\text{r,IL},n}\ r), & 0 \leq r \leq r_2 \\ C_3 J_0(\tilde{k}_{\text{r,IL},n}\ r) + C_4 Y_0(\tilde{k}_{\text{r,IL},n}\ r), & r_2 \leq r \leq r_3 \\ C_5 J_0(\tilde{k}_{\text{r,IL},n}\ r) + C_6 Y_0(\tilde{k}_{\text{r,IL},n}\ r), & r_3 \leq r \leq r_4 \end{cases}\]  
(9)

where \(J_0, Y_0\) denotes the 0th order Bessel and Neumann functions, respectively; \(C_1 - C_6\) the coefficients related by the following four boundary conditions at \(r = 0, r_2, r_3,\) and \(r_4:\)

1. At \(r = 0,\) the pressure is finite, thus equation (4) yields

\[C_2 = 0.\]  
(10)

2. From equations (6), (7), and (9), the rigid wall boundary condition at \(r = r_4\) gives

\[C_5 J_1(\tilde{k}_{\text{r,IL},n}\ r_4) + C_6 Y_1(\tilde{k}_{\text{r,IL},n}\ r_4) = 0\]  
(11)

with \(J_1\) and \(Y_1\) being the 1st order Bessel and Neumann functions, respectively.

3. From equations (6), (7) and (9), the continuity of radial particle velocity at \(r = r_2\) and \(r_3\) gives
\[
\frac{\dot{\rho}_{r_{ua}}}{\rho_{u_{a}}} [C_1 J_1(\tilde{k}_{r_{ua},r_2}) + C_2 Y_1(\tilde{k}_{r_{ua},r_2})] = \frac{\dot{\tilde{k}}_{r_{ua}}}{\dot{\rho}_{r_{ua}}} [C_3 J_1(\tilde{k}_{r_{ua},r_3}) + C_4 Y_1(\tilde{k}_{r_{ua},r_3})],
\]
(12)

\[
\frac{\dot{\tilde{k}}_{r_{ua}}}{\dot{\rho}_{r_{ua}}} [C_3 J_1(\tilde{k}_{r_{ua},r_3}) + C_4 Y_1(\tilde{k}_{r_{ua},r_3})] = \frac{\dot{\rho}_{r_{ua}}}{\dot{\tilde{k}}_{r_{ua}}} [C_3 J_1(\tilde{k}_{r_{ua},r_3}) + C_4 Y_1(\tilde{k}_{r_{ua},r_3})],
\]
(13)

(4) At \( r = r_2 \) and \( r_3 \), the continuity of the acoustic pressure in equation (4) yields

\[
C_1 J_0(\tilde{k}_{r_{ua},r_2}) + C_2 Y_0(\tilde{k}_{r_{ua},r_2}) = C_3 J_0(\tilde{k}_{r_{ua},r_3}) + C_4 Y_0(\tilde{k}_{r_{ua},r_3}),
\]
(14)

\[
C_3 J_0(\tilde{k}_{r_{ua},r_3}) + C_4 Y_0(\tilde{k}_{r_{ua},r_3}) = C_5 J_0(\tilde{k}_{r_{ua},r_3}) + C_6 Y_0(\tilde{k}_{r_{ua},r_3}).
\]
(15)

By assuming

\[
C_1 = 1,
\]
(16)

equations (10), (12)–(15) yield

\[
C_3 D_1 = J_0(\tilde{k}_{r_{ua},r_2}) Y_1(\tilde{k}_{r_{ua},r_2}) - \frac{\dot{\rho}_{r_{ua}}}{\dot{\tilde{k}}_{r_{ua}}} J_1(\tilde{k}_{r_{ua},r_2}) Y_0(\tilde{k}_{r_{ua},r_2}),
\]
(17)

\[
C_4 D_1 = -J_0(\tilde{k}_{r_{ua},r_2}) J_1(\tilde{k}_{r_{ua},r_2}) + \frac{\dot{\rho}_{r_{ua}}}{\dot{\tilde{k}}_{r_{ua}}} J_1(\tilde{k}_{r_{ua},r_2}) J_0(\tilde{k}_{r_{ua},r_2}),
\]
(18)

\[
C_3 D_2 = Y_1(\tilde{k}_{r_{ua},r_2}) \left[ J_0(\tilde{k}_{r_{ua},r_2}) Y_1(\tilde{k}_{r_{ua},r_2}) - \frac{\dot{\rho}_{r_{ua}}}{\dot{\tilde{k}}_{r_{ua}}} J_1(\tilde{k}_{r_{ua},r_2}) Y_0(\tilde{k}_{r_{ua},r_2}) \right]
\]
 \[- Y_0(\tilde{k}_{r_{ua},r_2}) \left[ -J_0(\tilde{k}_{r_{ua},r_2}) J_1(\tilde{k}_{r_{ua},r_2}) + \frac{\dot{\rho}_{r_{ua}}}{\dot{\tilde{k}}_{r_{ua}}} J_1(\tilde{k}_{r_{ua},r_2}) J_0(\tilde{k}_{r_{ua},r_2}) \right],
\]
(19)

\[
C_6 D_2 = -J_1(\tilde{k}_{r_{ua},r_2}) \left[ J_0(\tilde{k}_{r_{ua},r_2}) Y_1(\tilde{k}_{r_{ua},r_2}) - \frac{\dot{\rho}_{r_{ua}}}{\dot{\tilde{k}}_{r_{ua}}} J_1(\tilde{k}_{r_{ua},r_2}) Y_0(\tilde{k}_{r_{ua},r_2}) \right]
\]
 \[+ J_0(\tilde{k}_{r_{ua},r_2}) \left[ -J_0(\tilde{k}_{r_{ua},r_2}) J_1(\tilde{k}_{r_{ua},r_2}) + \frac{\dot{\rho}_{r_{ua}}}{\dot{\tilde{k}}_{r_{ua}}} J_1(\tilde{k}_{r_{ua},r_2}) J_0(\tilde{k}_{r_{ua},r_2}) \right]
\]
(20)

and

\[
D_1 = J_0(\tilde{k}_{r_{ua},r_2}) Y_1(\tilde{k}_{r_{ua},r_2}) - J_1(\tilde{k}_{r_{ua},r_2}) Y_0(\tilde{k}_{r_{ua},r_2}),
\]

\[
D_2 = \left[ J_0(\tilde{k}_{r_{ua},r_2}) Y_1(\tilde{k}_{r_{ua},r_2}) - J_1(\tilde{k}_{r_{ua},r_2}) Y_0(\tilde{k}_{r_{ua},r_2}) \right] \left[ J_0(\tilde{k}_{r_{ua},r_3}) Y_1(\tilde{k}_{r_{ua},r_3}) \right]
\]
 \[- J_1(\tilde{k}_{r_{ua},r_3}) Y_0(\tilde{k}_{r_{ua},r_3}) \]

The combination of equations (11), (19), and (20) leads to the characteristic function.
\[
J_1(\tilde{k}_{r,L,n} r_2) Y_1(\tilde{k}_{r,L,n} r_3) \left[ J_0(k_{r,L,n} r_2) Y_1(k_{r,L,n} r_3) - \frac{\tilde{\rho}_{L,n} k_{r,L,n}}{\rho_{L,n}} J_1(k_{r,L,n} r_2) Y_0(k_{r,L,n} r_3) \right] \\
- J_1(\tilde{k}_{r,L,n} r_4) Y_0(\tilde{k}_{r,L,n} r_3) \left[ -J_0(k_{r,L,n} r_2) J_1(k_{r,L,n} r_3) + \frac{\tilde{\rho}_{L,n} k_{r,L,n}}{\rho_{L,n}} J_1(k_{r,L,n} r_2) J_0(k_{r,L,n} r_3) \right] \\
= Y_1(\tilde{k}_{r,L,n} r_4) J_1(\tilde{k}_{r,L,n} r_3) \left[ J_0(k_{r,L,n} r_2) Y_1(k_{r,L,n} r_3) - \frac{\tilde{\rho}_{L,n} k_{r,L,n}}{\rho_{L,n}} J_1(k_{r,L,n} r_2) Y_0(k_{r,L,n} r_3) \right] \\
- Y_1(\tilde{k}_{r,L,n} r_4) J_0(\tilde{k}_{r,L,n} r_3) \left[ -J_0(k_{r,L,n} r_2) J_1(k_{r,L,n} r_3) + \frac{\tilde{\rho}_{L,n} k_{r,L,n}}{\rho_{L,n}} J_1(k_{r,L,n} r_2) J_0(k_{r,L,n} r_3) \right].
\]

(21)

In view of equation (3), equation (21) gives the solution for the eigenvalue (axial wave number \(k_{r,L,n}\)) for a given frequency; which then yields the solution of \(C_3 - C_6\) from equations (16)–(19), as well as the eigenfunctions for the axial and radial particle velocities, and pressure from equations (7e), (7f), and equation (9).

2.2 Wave propagation for the inlet/outlet pipe (Domain I and III)

In the inlet/outlet pipes, the acoustic pressure and axial particle velocity in the air are expressed as

\[
P_A(r,x) = \sum_{n=0}^{\infty} \left( A^n_+ e^{-j k_{r,A,n} x} + A^n_- e^{j k_{r,A,n} x} \right) \psi_{A,n}(r), \tag{22a}
\]

\[
u_{x,A}(r,x) = \frac{1}{\rho_0 c} \sum_{n=0}^{\infty} k_{r,A,n} \left( A^n_+ e^{-j k_{r,A,n} x} - A^n_- e^{j k_{r,A,n} x} \right) \psi_{A,n}(r), \tag{22b}
\]

\[
P_C(r,x) = \sum_{n=0}^{\infty} \left( C^n_+ e^{-j k_{r,C,n} (x-L)} + C^n_- e^{j k_{r,C,n} (x-L)} \right) \psi_{C,n}(r), \tag{22c}
\]

and

\[
u_{x,C}(r,x) = \frac{1}{\rho_0 c} \sum_{n=0}^{\infty} k_{r,C,n} \left( C^n_+ e^{-j k_{r,C,n} (x-L)} - C^n_- e^{j k_{r,C,n} (x-L)} \right) \psi_{C,n}(r), \tag{22d}
\]

where \(A^n_+\), \(A^n_-\), \(C^n_+\), and \(C^n_-\) are the modal amplitudes corresponding to components travelling in the positive and negative \(x\) directions, respectively; subscripts \(A\) and \(C\) denote domains I and III, respectively. The eigenfunctions are expressed as

\[
\psi_{A,n}(r) = J_0(k_{r,A,n} r), \tag{23a}
\]

\[
\psi_{C,n}(r) = J_0(k_{r,C,n} r) \tag{23b}
\]
with \( k_{r_A,n} \) and \( k_{r_C,n} \) the radial wave numbers satisfying the rigid wall boundary condition of

\[
J_1(k_{r_A,n} r_1) = 0, \quad (24a)
\]

\[
J_1(k_{r_C,n} r_1) = 0; \quad (24b)
\]

and the wave numbers in the axial direction are

\[
k_{x,A,n} = \begin{cases} 
\sqrt{k_0^2 - k_{r_A,n}^2}, & k_0 > k_{r_A,n} \\
-\sqrt{k_0^2 - k_{r_A,n}^2}, & k_0 < k_{r_A,n}
\end{cases}, \quad (25a)
\]

\[
k_{x,C,n} = \begin{cases} 
\sqrt{k_0^2 - k_{r_C,n}^2}, & k_0 > k_{r_C,n} \\
-\sqrt{k_0^2 - k_{r_C,n}^2}, & k_0 < k_{r_C,n}
\end{cases}, \quad (25b)
\]

with \( k_0 = \frac{2\pi}{c_0} \).

2.3 Transmission loss prediction

In view of the pressure and velocity expressed in equations (4), (5), and (22), the continuities of the pressure and velocity across the interface

\[
P_A = P_B, \quad 0 \leq r \leq r_1, x = 0, \quad (26a)
\]

\[
u_{x,A} = \begin{cases} 
u_{x,A}, & \text{for } 0 \leq r \leq r_1, x = 0 \\
0, & \text{for } r_1 \leq r \leq r_3, x = 0 \end{cases}, \quad (26b)
\]

\[
P_C = P_B, \quad 0 \leq r \leq r_1, x = L, \quad (26c)
\]

\[
u_{x,B} = \begin{cases} 
u_{x,C}, & \text{for } 0 \leq r \leq r_1, x = L \\
0, & \text{for } r_1 \leq r \leq r_3, x = L \end{cases}, \quad (26d)
\]

yield

\[
\sum_{n=0}^{\infty} (A_n^+ + A_n^-) \psi_{A,n}(r) = \sum_{n=0}^{\infty} (B_n^+ + B_n^-) \psi_{B,n,p}(r), \quad \text{for } 0 \leq r \leq r_1, \quad (27a)
\]

\[
\sum_{n=0}^{\infty} k_{x,B,n} (B_n^+ - B_n^-) \psi_{B,n,p}(r) = \begin{cases} 
\sum_{n=0}^{\infty} k_{x,A,n} (A_n^+ - A_n^-) \psi_{A,n}(r), & \text{for } 0 \leq r \leq r_1 \\
0, & \text{for } r_1 \leq r \leq r_3 \end{cases}, \quad (27b)
\]

\[
\sum_{n=0}^{\infty} (C_n^+ + C_n^-) \psi_{C,n}(r) = \sum_{n=0}^{\infty} (B_n^+ e^{-\rho_{k,s,x} L} + B_n^- e^{\rho_{k,s,x} L}) \psi_{B,n,p}(r), \quad \text{for } 0 \leq r \leq r_1, \quad (27c)
\]
\[ \sum_{n=0}^{\infty} k_{x,B,n} \left( B_{n}^+ e^{-jk_{x,B,n}L} - B_{n}^- e^{jk_{x,B,n}L} \right) \psi_{B,n,x} \left( r \right) \]

\[ = \left\{ \begin{array}{ll}
\sum_{n=0}^{\infty} k_{x,C,n} \left( C_{n}^+ - C_{n}^- \right) \psi_{C,n} \left( r \right), & \text{for } 0 \leq r \leq r_1 \\
0 & \text{for } r_1 \leq r \leq r_4
\end{array} \right. \]  

Using the approach proposed in Xu et al. (2004), the wave amplitudes \( A_{n}^+ \), \( A_{n}^- \), \( B_{n}^+ \), \( B_{n}^- \), \( C_{n}^+ \) and \( C_{n}^- \) in equation (27) can be determined with the assumption of \( A_{0}^+ = 1, A_{0,1,2,\ldots}^- = 0 \), and \( C_{0,1,2,\ldots}^- = 0 \). Then, assuming that the transmitted waves in the outlet pipe are non-propagating modes except the first mode with \( C_{0}^+ \), the transmission loss is determined as

\[ TL = -20 \log_{10} \left| C_{0}^+ \right| \]  

(28)

3 Results and discussion

The characteristic impedance \( \tilde{Z} = \rho \delta \) and the wave number \( \tilde{k} \) of the absorbing material in the present study are given as (Nice, 2000)

\[ \frac{\tilde{Z}}{Z_0} = [1 + 0.0855(f/R)^{-0.754}] + j[-0.0765(f/R)^{-0.732}] \]  

(29a)

\[ \frac{\tilde{k}}{k_0} = [1 + 0.1472(f/R)^{-0.577}] + j[-0.1734(f/R)^{-0.595}] \]  

(29b)

with \( Z_0 = \rho_0 \delta_0 \) being the characteristic impedance of the air, and \( f [\text{Hz}] \) denotes frequency and \( R [\text{mks Rayls/m}] \), the resistivity of the absorbing material. Parameters of the baseline configuration are: \( L = 25.72 \text{ cm}, \ r_1 = r_2 = 2.45 \text{ cm}, \ r_3 = 5 \text{ cm}, \ r_4 = 8.22 \text{ cm}, \ R_{n_x} = 0 \text{ Rayls/m (without fibre)}, \ R_{n_y} = 4,896 \text{ Rayls/m (filling density being 100 g/l)}, \ R_{n_z} = 17,378 \text{ Rayls/m (filling density being 200 g/l).} \) The selected densities and therefore the associated resistances are typical of the range used in filled silencers.

The transmission loss results from the analytical approach are compared with 3-D FEM predictions (SYSNOISE, 2001) in Figure 2 for two expansion chambers with different arrangements of absorbing material. For the current geometry and frequency range of interest, the analytical results agree well with the FEM predictions for both configurations (the maximum difference of transmission loss is about 1 dB); and the computational time for the analytical method is less than 2% that of FEM.
Figure 2  Transmission loss of a layered dissipative expansion chamber
\((L = 25.72 \, \text{cm}, r_1 = r_2 = 2.45 \, \text{cm}, r_3 = 5 \, \text{cm}, r_4 = 8.22 \, \text{cm})\)

Following Cumming's discussion on the modulus of radial eigenfunctions in a dissipative silencer (Cummings and Chang, 1988), Figures 3(a) and (b) present the normalised modulus of eigenfunctions of pressure, \(|\psi_{B,n,r}| [\text{equation (9)}]\), in layered dissipative chambers for

(a)  \(R_{H_a} = 4,896 \, \text{Rayls/m}, \ R_{H_c} = 17,378 \, \text{Rayls/m}\)

(b)  \(R_{H_a} = 17,378 \, \text{Rayls/m}, \ R_{H_c} = 4,896 \, \text{Rayls/m}\).

The location \(r_2/r_4 = 0.2981\) corresponds to the interface of domains \(\Pi_a\) and \(\Pi_b\); and \(r_2/r_3 = 0.6083\) to that of \(\Pi_b\) and \(\Pi_c\). For case (a), \(|\psi_{B,n,r}|\) is maximum at the rigid wall \((r = r_4)\) for \(n = 0\); however, for \(n > 0\), the maxima are at \(r = 0\). The number of the minima of \(|\psi_{B,n,r}|\) is equal to the order of the wave mode. For all wave modes, the slope of the pressure is zero at both the centre line \((r = 0)\) and the rigid wall \((r = r_4)\). The changes in acoustic properties introduce slope discontinuities at both \(r = r_2\) and \(r = r_3\). The results for case (b) are similar to that of case (a) except that, for \(n = 0\), \(|\psi_{B,n,r}|\) reaches a maximum at a position between \(r_2\) and \(r_3\); and the slope discontinuity at \(r = r_2\) in case (b) is more distinct due to stronger variation in the acoustic properties.
Figure 3  Normalised modulus of eigenfunctions of pressure $|\psi_{B,n,P}|$ in the layered dissipative expansion chamber at 3,200 Hz ($r_2 = 2.45$ cm, $r_3 = 5$ cm, $r_4 = 8.22$ cm):
(a) $R_{n_{l_{2}}} = 4,896$ Rayls/m and $R_{n_{l_{3}}} = 17,378$ Rayls/m and (b) $R_{n_{l_{2}}} = 17,378$ Rayls/m and $R_{n_{l_{3}}} = 4,896$ Rayls/m
Analytical predictions of transmission loss for layered dissipative silencers with a different arrangement of flow resistivities are shown in Figure 4, including silencers with:

(a) $R_{h_b} = 4,896$ Rayls/m and $R_{h_c} = 17,378$ Rayls/m

(b) $R_{h_b} = 17,378$ Rayls/m and $R_{h_c} = 4896$ Rayls/m

(c) $R_{h_b} = R_{h_c} = 4,896$ Rayls/m

(d) $R_{h_b} = R_{h_c} = 17,378$ Rayls/m; [thus (c) and (d) representing the limits with one or the other resistivity throughout]

(e) uniform flow resistivity of 6.913 Rayls/m [the corresponding filling density is $((5^2 - 2.45^2) \times 200 + (8.22^2 - 5^2) \times 100)/(8.22^2 - 2.45^2) = 131$ g/l, the total mass of the fibre being the same as the silencer (a)] from $r_2$ to $r_4$

(f) 11,989 Rayls/m [the corresponding density is $((5^2 - 2.45^2) \times 100 + (8.22^2 - 5^2) \times 200)/(8.22^2 - 2.45^2) = 169$ g/l, the total mass of the fibre being the same as the silencer (b)] again from $r_2$ to $r_4$.

The dimensions of the baseline configuration have been retained here, while modifying the filling densities. As expected, the performance of the silencer with high flow resistivity ($R_{h_b} = R_{h_c} = 11,989$ or 17,378 Rayls/m) is better from mid to high frequencies, but worse at lower frequencies (below about 250 Hz), than that of the silencer with low flow resistivity ($R_{h_b} = R_{h_c} = 4,896$ or 6,913 Rayls/m). Compared to the silencer with the same total amount of fibre and uniform distribution of flow resistivity (the silencer with $R_{h_b} = R_{h_c} = 11,989$ Rayls/m), the layered silencer (a) (with the filling density of inner layer less than that of the outer one) exhibits a modest increase in transmission loss at the mid-frequency range (1,300 Hz < $f$ < 2,500 Hz), followed by a decrease at higher frequencies (above 2,500 Hz). With fibre of low density in the outer layer, the transmission loss of the layered silencer (b) is slightly reduced at frequencies below 2,100 Hz, followed by a modest improvement at frequencies above 2,100 Hz as compared to the silencer of uniform flow resistivity 6,913 Rayls/m. The transmission loss of the layered silencer (a) is higher than (b) primarily due to the effective (average) density difference. Figure 5 zooms into the low frequency region of Figure 4. Note the reversal in the trends of lowest (c) vs. highest (d) uniform density cases.
Figure 4 Transmission loss prediction for layered dissipative expansion chambers with different flow resistivities \((L = 25.72 \text{ cm}, r_1 = r_2 = 2.45 \text{ cm}, r_3 = 5 \text{ cm}, r_4 = 8.22 \text{ cm})\)

\[ R_{in} = 4896 \text{ Rayls/m}, R_{in} = 17378 \text{ Rayls/m}; \]
\[ R_{in} = 17378 \text{ Rayls/m}, R_{in} = 4896 \text{ Rayls/m}; \]
\[ R_{in} = R_{in} = 4896 \text{ Rayls/m}; \]
\[ R_{in} = R_{in} = 17378 \text{ Rayls/m}; \]
\[ R_{in} = R_{in} = 6913 \text{ Rayls/m}; \]
\[ R_{in} = R_{in} = 11989 \text{ Rayls/m}. \]

Figure 5 Expanded low frequency behaviour of Figure 4 (the legend is the same as in Figure 4)

Figure 6 illustrates the influence of \(r_3\) on transmission loss. The filling densities of the baseline configuration have been retained here, while modifying only the location of the interface between two fibre layers. Generally, increasing \(r_3\) (therefore increasing the thickness of the fibre layer with lower flow resistivity and decreasing that of higher resistivity) shows a slight improvement at low frequencies similar to Figure 5, while reducing the transmission loss at mid-frequency (below the peak) range considerably. In addition, the transmission loss of the silencer with smaller \(r_3\) reaches its peak at relatively lower frequency.
Figure 6  Transmission loss prediction for layered dissipative expansion chambers with different $r_3$ ($L = 25.72$ cm, $r_1 = r_2 = 2.45$ cm, $r_4 = 8.22$ cm, $R_{||} = 4,896$ Rayls/m, $R_{\perp} = 17,378$ Rayls/m)

The analytical predictions of transmission loss with various chamber lengths ($L = 15$, $25.72$, $35$, and $40$ cm) are depicted in Figure 7. As expected, increasing $L$ improves the sound attenuation of the layered dissipative chamber at all frequencies.

Figure 7  Transmission loss prediction for layered dissipative expansion chambers with different $L$ ($r_1 = r_2 = 2.45$ cm, $r_3 = 5$ cm, $r_4 = 8.22$ cm, $R_{||} = 4,896$ Rayls/m, $R_{\perp} = 17,378$ Rayls/m):

$L = 15$ cm.
$L = 25.72$ cm.
$L = 35$ cm.
$L = 45$ cm.
4 Conclusions

An analytical approach has been developed to investigate the acoustic attenuation of a layered dissipative silencer, which is a single-pass, axisymmetric, cylindrical expansion chamber lined with two concentric annular layers of fibrous material with different resistance. The transmission loss is obtained analytically by applying continuities of the pressure/axial particle velocity at the interfaces of the expansion/contraction. The effects of the chamber length and density arrangement of the fibre on the performance of this layered dissipative chamber have been investigated.

Results reveal that lower flow resistivity is generally desirable in the inner layer coupled with a higher resistivity in the outer layer, which helps improve the transmission loss from mid to high frequencies, without significant deterioration at low frequencies. The determining factor behind this behaviour is related to the overall average density of the fibrous material. The dissipative expansion chambers with multiple layers of absorbing material and perforated screens are being investigated next.

References


