Effect of voids on the acoustics of perforated dissipative silencers

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Abstract: The effect of voids on the acoustic performance of a perforated dissipative expansion chamber is investigated by axially staggering filled/empty segments external to the perforations. A two-dimensional analytical approach is used to determine the Transmission Loss (TL) through pressure and velocity matching at the boundaries of the inlet/outlet ducts, rigid end-plates, and the interfaces between the fibrous material and void. The analytical results are then compared with the experiments and numerical predictions for one specific configuration, showing a reasonable agreement. The effects of void length and location, and the properties of fibrous material on the acoustic attenuation of this perforated dissipative expansion chamber are also discussed.

Keywords: voids effects in perforated dissipative silencers.


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1 Introduction

Dissipative silencers, usually in the form of expansion chambers filled with fibrous material, are widely used to suppress noise in automotive exhaust and ventilation systems due to their broadband attenuation characteristics and low back pressure. In order to investigate their acoustic performance, both three-dimensional (3D) boundary element (BEM) (Selamet et al., 2001; SYSNOISE, 2001) and Finite Element Methods (FEM) (SYSNOISE, 2001; Peat and Rath, 1995; Astley and Cummings, 1987) are employed for irregular shapes, whereas two-dimensional (2D) numerical techniques (Cummings, 1995; Glav, 1996, 2000) are adopted for silencers with axially uniform and arbitrary cross sections. In addition to these numerical techniques, various analytical approaches (Cummings and Chang, 1988; Peat, 1991; Auregan et al., 2001; Xu et al., 2004; Selamet et al., 2004) have also been employed for symmetric dissipative silencers, such as those with cylindrical chambers.

To improve the acoustic performance of dissipative silencers, configurations beyond these simple dissipative expansion chambers have also been examined. Selamet et al. (2003) designed and analysed a dissipative-reactive (hybrid) silencer consisting of two dissipative expansion chambers combined with a reactive component (a Helmholtz resonator) between them. Their numerical investigation demonstrated that the reactive component was able to enhance the acoustic performance of this hybrid silencer at low frequencies. Selamet et al. (2005a) investigated the effect of inlet/outlet extensions on the acoustic attenuation of a dissipative circular expansion chamber. Selamet et al. (2005b) proceeded to study the acoustic performance of a layered dissipative silencer, which is a single-pass, concentric, cylindrical expansion chamber lined with concentric annular fibre layers of different resistance. The lower flow resistivity is found to be generally desirable in the inner layer coupled with a higher resistivity in the outer layer, which improves the TL from mid to high frequencies, without significant deterioration at low frequencies. Huff (2005) investigated experimentally the impact of fibrous material filling density variation on the acoustic performance of a perforated dissipative silencer.
chamber. A simple cylindrical silencer with a perforated duct has been used to experimentally investigate the impact of large density variations and voids, in both the radial and axial directions, upon the acoustic performance of a silencer. To improve the acoustic attenuation of dissipative silencers, Munjal (2003) investigated a pod silencer, which is a lined circular duct with a cylindrical pod inside. Other similar concepts, such as splitter-silencers (Ko, 1975; Cummings and Sormaz, 1993), parallel-baffle mufflers (Mechel, 1990a, 1990b), and bar-silencers (Nilsson and Söderqvist, 1983; Cummings and Astley, 1996) were also considered to improve the noise attenuation.

Other than the concentric layers of fibrous materials of Selamet et al. (2005b), and the fibrous material of varying density of Huff (2005), the absorbent in the expansion chambers discussed thus far is assumed to be evenly filled. There are circumstances where the fibre distribution may not be even leading potentially to voids, or the silencers may be intentionally designed with voids to reduce the cost and weight particularly when partial filling provides a sufficient noise attenuation. Therefore, it is desirable to investigate the influence of the voids on the acoustic performance of silencers. By introducing the voids in the form of annular segments attached to perforations, the present study:

- investigates theoretically, numerically, and experimentally the acoustic attenuation of a perforated dissipative expansion chamber
- examines the effect of void length and location, and absorbent distribution on the acoustic attenuation behaviour.

A 2D, closed-form analytical solution is developed to investigate the acoustic performance. The results are then compared to the experimental data for one configuration, and the predictions from a 3D numerical method (BEM). Following this introduction, Section 2 obtains the TL by a two-dimensional analytical approach in terms of a pressure/velocity matching technique. Section 3 discusses the effect of void length, void location, and absorbent distribution on the TL. The study is concluded with final remarks in Section 4.

2 Analytical approach

Consider a cylindrical dissipative chamber, shown in Figure 1, of length $L$ and radius $r_2$, with a number of different annular layers of sound-absorbing material ($L_1, L_2, ..., L_m, ..., L_M$; void is a special case with the fibrous material removed) placed in the expansion chamber. The inlet and outlet ducts of radius $r_1$ are assigned domains I and III, and the chamber $\Pi_{(a,b),(1,...,m,..,M)}$ with subscript $a$ designating the domain in the central airway $0 \leq r \leq r_1$, $b$ the domain $r_1 \leq r \leq r_2$, and $m$ an annular layer in the axial direction. A perforated screen with porosity $\phi$ is located at $r = r_1$ to separate the central airway from the absorbing material and voids in the expansion chamber. The absorbing material for the $m$th ($m = 1, ..., M$) layer is assumed to be homogeneous and isotropic, characterised by the complex speed of sound $\tilde{c}_m$ and density $\tilde{\rho}_m$ (The expressions are deferred to Appendix A).
2.1 Wave propagation in the expansion chamber (Domain II)

Consider the central perforated dissipative expansion chamber of radius $r_2$ (domain II in Figure 1) with a perforated screen of radius $r_1$ and porosity $\phi$ that separates domains $\Pi_a$ and $\Pi_b$. Domain $\Pi_m$ with $x_m \leq x \leq x_{m+1}$ is used as an example to illustrate the wave propagation characteristics in the perforated dissipative expansion chamber. The wave characteristics in other domains of the chamber (domains II_1, ..., II_{m-1}, II_{m+1}, ..., II_M) can be obtained similarly.

The pressure for domain $\Pi_m$ may then be expressed as (Selamet et al., 2004):

$$P_{B_n}(r, x) = \sum_{n=0}^{\infty} \left( B_{m,n}^+ e^{-j k_{x_B,n} (x-x_m)} + B_{m,n}^- e^{j k_{x_B,n} (x-x_m)} \right) \psi_{B_n,n,p}(r),$$  \hspace{1cm} (1)

where subscripts $B_m$ denote domain II; $B_{m,n}^+$ and $B_{m,n}^-$ the modal amplitudes; $k_{x_B,n}$ the axial wave number for both the central airway and absorbent material. The transverse modal eigenfunction for the pressure $\psi_{B_n,n,p}(r)$ is expressed as (Selamet et al., 2004):

$$\psi_{B_n,n,p}(r) = \begin{cases} 
J_0(k_{r,B_n,n}r), & 0 \leq r \leq r_1 \\
Y_1(\tilde{k}_{r,B_n,n}r_2) \left[ J_0(k_{r,B_n,n}r_1) + \frac{j\tilde{z}_{B_n} k_{r,B_n,n}}{k_0} J_1(k_{r,B_n,n}r_1) \right] \\
J_0(\tilde{k}_{r,B_n,n}r_1)Y_1(\tilde{k}_{r,B_n,n}r_2) - J_1(\tilde{k}_{r,B_n,n}r_2)Y_0(\tilde{k}_{r,B_n,n}r_1) \\
n \times J_0(\tilde{k}_{r,B_n,n}r) - \frac{J_1(\tilde{k}_{r,B_n,n}r_2)}{Y_0(\tilde{k}_{r,B_n,n}r_2)} Y_0(\tilde{k}_{r,B_n,n}r) 
\end{cases}, \hspace{1cm} r_1 \leq r \leq r_2, \hspace{1cm} (2)
where $\tilde{\xi}$ (expression is given in Appendix B) is the non-dimensionalised perforate acoustic impedance relating the acoustic pressure in the inner duct and outer chamber through the interface; $k_{r,B_n,n}$ and $\tilde{k}_{r,B_n,n}$ are the radial wave numbers for the air and fibrous material, respectively, which are related to the axial wave number $k_{x,B_n,n}$ by

$$k_{r,B_n,n}^2 + k_{x,B_n,n}^2 = k_0^2 = (\omega / c_0)^2,$$  \hspace{1cm} (3a)$$

and

$$\tilde{k}_{r,B_n,n}^2 + k_{x,B_n,n}^2 = \tilde{k}_m^2 = (\omega / \tilde{c}_m)^2,$$  \hspace{1cm} (3b)$$

where $\omega$ is the radial frequency, and $c_0$ being the speed of sound in the air.

The wave numbers can be solved from the characteristic equation (derived from the boundary conditions of acoustic pressure and radial particle velocity at $r = 0$, $r_1$, and $r_2$ (Selamet et al., 2004)

$$\frac{\rho_0 \tilde{k}_{r,B_n,n}}{\tilde{\rho} k_{r,B_n,n}} \left[ \frac{J_0(k_{r,B_n,n} r_1)}{J_1(k_{r,B_n,n} r_1)} + \frac{j \tilde{\xi}_{B_n} k_{r,B_n,n}}{k_0} \right] = \frac{J_0(\tilde{k}_{r,B_n,n} r_1) Y_1(\tilde{k}_{r,B_n,n} r_2) - Y_0(\tilde{k}_{r,B_n,n} r_1) J_1(\tilde{k}_{r,B_n,n} r_2)}{J_1(\tilde{k}_{r,B_n,n} r_1) Y_1(\tilde{k}_{r,B_n,n} r_2) - Y_1(\tilde{k}_{r,B_n,n} r_1) J_1(\tilde{k}_{r,B_n,n} r_2)},$$

(4)

In view of equation (1), the linearised momentum equation gives the particle velocity in the axial direction as

$$u_{B_n}(r,x) = \frac{1}{\rho_0 \omega} \sum_{n=0}^{\infty} k_{x,B_n,n} \left( B_{m,n}^+ e^{-j k_{x,B_n,n} (x-x_n)} - B_{m,n}^- e^{j k_{x,B_n,n} (x-x_n)} \right) \psi_{B_n,n,r}(r)$$

(5)

with the transverse modal eigenfunction for the velocity in the axial direction being expressed as

$$\psi_{B_n,n,r}(r) = \begin{cases} 
J_0(k_{r,B_n,n} r), & 0 \leq r \leq r_1 \\
Y_1(\tilde{k}_{r,B_n,n} r_1) \left[ \frac{J_0(k_{r,B_n,n} r_1)}{k_0} + \frac{j \tilde{\xi}_{B_n} k_{r,B_n,n}}{k_0} \right], & r_1 \leq r \leq r_2 \\
J_0(\tilde{k}_{r,B_n,n} r_1) Y_1(\tilde{k}_{r,B_n,n} r_2) - J_1(\tilde{k}_{r,B_n,n} r_1) Y_0(\tilde{k}_{r,B_n,n} r_1), & r_2 \leq r \leq r_3 \\
\times \frac{\rho_0}{\tilde{\rho}_m} \left[ J_0(\tilde{k}_{r,B_n,n} r) - \frac{J_1(\tilde{k}_{r,B_n,n} r_2)}{Y_1(\tilde{k}_{r,B_n,n} r_2) Y_0(\tilde{k}_{r,B_n,n} r_2)} \right], & r_3 \leq r \leq r_4.
\end{cases}$$

(6)

Similar to equations (1) and (5), the pressure and the axial particle velocity for all other layers in domain II can be determined.
2.2 Wave propagation in the inlet/outlet pipe (Domains I and III)

The acoustic pressures in the inlet/outlet pipes are expressed as

\[ P_A(r,x) = \sum_{n=0}^{\infty} \left( A_n^+ e^{-j\beta_{x,AA} x} + A_n^- e^{j\beta_{x,AA} x} \right) \psi_{A,n}(r), \]

\[ P_C(r,x) = \sum_{n=0}^{\infty} \left( C_n^+ e^{-j\beta_{x,CA}(x-L)} + C_n^- e^{j\beta_{x,CA}(x-L)} \right) \psi_{C,n}(r), \]

where \( A_n^+ \), \( A_n^- \), \( C_n^+ \) and \( C_n^- \) are the modal amplitudes corresponding to components travelling in the positive and negative \( x \) directions, respectively; subscripts \( A \) and \( C \) denote domains I and III, respectively. The eigenfunctions are expressed as

\[ \psi_{A,n}(r) = J_0(k_{r,AA} r), \]

\[ \psi_{C,n}(r) = J_0(k_{r,CA} r) \]

with \( k_{r,AA} \) and \( k_{r,CA} \) being the radial wave numbers satisfying the rigid wall boundary condition of

\[ J_1(k_{r,AA} r_1) = 0, \]

\[ J_1(k_{r,CA} r_1) = 0; \]

the radial wave number is related to the axial wave numbers by

\[ k_{x,AA}^2 + k_{r,AA}^2 = k_0^2 = (\omega / c_0)^2, \]

\[ k_{x,CA}^2 + k_{r,CA}^2 = k_0^2 = (\omega / c_0)^2. \]

From the linearised momentum equation, equation (7) gives the expressions of the axial particle velocity in the inlet/outlet pipes as

\[ u_A(r,x) = \frac{1}{\rho_0 \omega} \sum_{n=0}^{\infty} k_{x,AA} \left( A_n^+ e^{-j\beta_{x,AA} x} - A_n^- e^{j\beta_{x,AA} x} \right) \psi_{A,n}(r), \]

\[ u_C(r,x) = \frac{1}{\rho_0 \omega} \sum_{n=0}^{\infty} k_{x,CA} \left( C_n^+ e^{-j\beta_{x,CA}(x-L)} - C_n^- e^{j\beta_{x,CA}(x-L)} \right) \psi_{C,n}(r). \]

2.3 Transmission loss (TL) prediction

With the pressure and axial particle velocity expressed in equations (1), (5), (7), and (11), TL can then be obtained by solving the unknown coefficients \( A_n \), \( B_{m,n} \) (with \( m = 1, 2, \ldots, M \)), and \( C_n \) using the pressure/velocity boundary conditions at the interfaces of \( x = 0, x_m \) (with \( m = 2, 3, \ldots, M \)), and \( L \). At the interfaces of the expansion and contraction, the acoustic pressure and axial particle velocity continuity conditions reveal
\[ P_A = P_{B_1} , \]  \( 0 \leq r \leq r_1, \ x = 0, \) \( \) (12a)

\[ u_n = \begin{cases} u_A, & \text{for } 0 \leq r \leq r_1, \ x = 0 \\ 0, & \text{for } r_1 \leq r \leq r_2, \ x = 0 \end{cases}, \] \( \) (12b)

\[ P_c = P_{B_2}, \]  \( 0 \leq r \leq r_1, \ x = L, \) \( \) (12c)

\[ u_n = \begin{cases} u_c, & \text{for } 0 \leq r \leq r_1, \ x = L \\ 0, & \text{for } r_1 \leq r \leq r_2, \ x = L \end{cases}, \] \( \) (12d)

At the interfaces of different fibrous material layers (\( x = x_m \) with \( m = 2, 3, \ldots, M \)), the continuities of acoustic pressure and axial particle velocity reveal

\[ P_{B_{m+1}} = P_{B_m}, \]  \( 0 \leq r \leq r_2, \ x = x_m \ (m = 2, 3, \ldots, M) \) \( \) (13a)

\[ u_{B_{m+1}} = u_{B_m}, \]  \( 0 \leq r \leq r_2, \ x = x_m \ (m = 2, 3, \ldots, M). \) \( \) (13b)

In view of equations (1), (5), (7), and (11), equations (12) and (13) yield

\[ \sum_{n=0}^{\infty} (A_n^+ + A_n^-) \psi_{A,n}(r) = \sum_{n=0}^{\infty} (B_{1,n}^+ + B_{1,n}^-) \psi_{B_{1,n},p}(r), \]  \( 0 \leq r \leq r_1, \) \( \) (14a)

\[ \sum_{n=0}^{\infty} k_{x,B_{1,n}} (B_{1,n}^+ - B_{1,n}^-) \psi_{B_{1,n},u}(r) = \begin{cases} \sum_{n=0}^{\infty} k_{x,A,n} (A_n^+ - A_n^-) \psi_{A,n}(r), & 0 \leq r \leq r_1 \\ 0, & r_1 \leq r \leq r_2 \end{cases}, \] \( \) (14b)

\[ \sum_{n=0}^{\infty} (C_n^+ + C_n^-) \psi_{C,n}(r) \]

\[ = \sum_{n=0}^{\infty} \left( B_{M,n}^+ e^{-jx_{B_M} \ell_M} + B_{M,n}^- e^{jx_{B_M} \ell_M} \right) \psi_{B_M,n,p}(r), \]  \( 0 \leq r \leq r_1, \) \( \) (14c)

\[ \sum_{n=0}^{\infty} k_{x,B_M,n} \left( B_{M,n}^+ e^{-jx_{B_M} \ell_M} - B_{M,n}^- e^{jx_{B_M} \ell_M} \right) \psi_{B_M,n,u}(r) \]

\[ = \begin{cases} \sum_{n=0}^{\infty} k_{x,C,n} (C_n^+ - C_n^-) \psi_{C,n}(r), & 0 \leq r \leq r_1 \\ 0, & r_1 \leq r \leq r_2 \end{cases}, \] \( \) (14d)

\[ \sum_{n=0}^{\infty} \left( B_{m,n}^+ e^{-jx_{B_m} \ell_m} + B_{m,n}^- e^{jx_{B_m} \ell_m} \right) \psi_{B_n,n,p}(r) \]

\[ = \sum_{n=0}^{\infty} (B_{m+1,n}^+ + B_{m+1,n}^-) \psi_{B_{m+1},n,p}(r), \]  \( 0 \leq r \leq r_2 \) and \( m = 1, 2, 3, \ldots, M - 1, \) \( \) (14e)

and

\[ \sum_{n=0}^{\infty} k_{x,B_{m+1,n}} \left( B_{m+1,n}^+ e^{-jx_{B_M} \ell_m} - B_{m+1,n}^- e^{jx_{B_M} \ell_m} \right) \psi_{B_{m+1},n,u}(r) \]

\[ = \sum_{n=0}^{\infty} k_{x,B_{m+1,n}} (B_{m+1,n}^+ - B_{m+1,n}^-) \psi_{B_{m+1},n,u}(r), \]  \( 0 \leq r \leq r_2 \) and \( m = 1, 2, 3, \ldots, M - 1. \) \( \) (14f)
With the assumption of $A_0^+ = 1, A_{1,2,3,\ldots}^+ = 0$, and $C_{0,1,2,\ldots}^- = 0$, the wave amplitudes $A_n^-, B_{1,n}^+, B_{2,n}^+, \ldots, B_{M,n}^-, B_{1,n}^-, B_{2,n}^-, \ldots, B_{M,n}^-$, and $C_n^+$ in equation (14) can then be determined using the approach in Xu et al. (2004). Then, assuming that the transmitted waves in the outlet pipe are non-propagating modes except the first mode with the amplitude of $C_0^+$, TL is determined as

$$TL = -20\log_{10}|C_0^+|.$$  \hspace{1cm} (15)

### 3 Results and discussion

A perforated dissipative expansion chamber with multiple annular absorbent layers is studied here (Recall Figure 1). Chosen to be consistent with Selamet et al. (2004), the geometry of the base configuration is specified as: $L = 25.72$ cm, $r_1 = 2.45$ cm, $r_2 = 8.22$ cm, porosity of the perforated duct $\phi = 8\%$, thickness of the perforated duct $t_w = 0.09$ cm, and diameter of the holes $d_h = 0.249$ cm. This baseline geometry is retained in all figures that follow. Absorbing materials with filling densities of 100 g/l and 200 g/l (with the complex speed of sound and density given in Appendix A) are considered.

A prototype has been fabricated with the foregoing dimensions and three annular layers: the first ($L_1$) and third ($L_3$) annular empty segments are separated by an absorbent of length ($L_2$) and density of 100 g/l; and the length ratio is chosen as $L_1 : L_2 : L_3 = 1 : 2 : 1$. Figure 2 compares its TL determined from analytical and numerical (BEM) approaches, and the experiments based on a two–microphone technique (Selamet et al., 2001). The analytical results agree well with the BEM predictions and show a reasonable comparison with the measurement. TL is obtained by using 3, 6, and 9 acoustic modes in the analytical approach. Results reveal that, for the current geometry and the frequency range of interest, the analytical predictions are accurate enough with six or more acoustic modes being included. Thus, only analytical results with nine acoustic modes have been presented from Figure 2 onwards.

**Figure 2** TL of a perforated dissipative expansion chamber with three axial-direction layers (the first and third layers are void, and the second is fibrous material with filling density of 100 g/l) with $L_1 : L_2 : L_3 = 1 : 2 : 1$: ——, Analytical; •, BEM; o, Experiment
Figure 3 investigates the effect of void length by presenting the TL of silencers with two annular layers: the first one of length $L_1$ is empty, and the second one of length $L_2$ is filled with fibrous material of 100 g/l. The results of two extreme cases – one fully filled with fibrous material and the other one empty – are also superimposed. Increasing the void length $L_1$ reduces the TL, as expected, since there is less absorptive material present in the chamber. However, it is important to note that for small voids, the reduction in TL is rather small. For example, in the case of $L_1 : L_2 = 1 : 15$ (or $L_1/L_2 = 1/16$), the reduction in peak TL is less than 2 dB. The influence of the void, regardless of size, is also not observable at low frequencies (below 250 Hz).

**Figure 3** Effect of void length on the TL of silencers with two axial-direction layers (the first layer is void, and the second is fibrous material with a filling density of 100 g/l): ———, No void; ———, $L_1 : L_2 = 1 : 15$; ..., $L_1 : L_2 = 2 : 14$; ———, $L_1 : L_2 = 4 : 12$; ———, $L_1 : L_2 = 8 : 8$;+, no fibre

The effect of void location on the TL is illustrated in Figure 4 by analysing three–annular–layer silencers: the first and third are filled with fibrous material of 100 g/l, and the second is empty at a fixed length. The maximum spread in TL with different void locations remains less than 1 dB. Therefore, the effect of location may be neglected for small voids.

The effect of fibrous material location on the TL is shown in Figure 5 for a silencer with three–annular–layers: the first and third are empty, and the second is filled with fibrous material of 100 g/l for a fixed length of half of the total expansion chamber. Note that the silencer with $L_1 : L_2 : L_3 = 4 : 8 : 4$ is identical to that of Figure 2. Except at high frequencies (above 2,500 Hz) and, to an extent, around the TL peak, the attenuation of silencers with the fibrous material placed in the middle of the expansion chamber appears to be higher than the others.
Figure 4  Effect of void location on the TL of silencers with three axial-direction layers (the first and third layers are fibrous material with filling density of 100 g/l, and the second void) with $L_1 : L_2 : L_3$ of: ——, 0 : 1 : 14; -----, 1 : 1 : 13; ..., 3 : 1 : 11; —— ——, 5 : 1 : 9; —— ——, 7 : 1 : 7

![Graph showing Transmission Loss vs Frequency for various void locations. The graph includes a legend with the void ratios for each line segment.]

Figure 5  Effect of location of fibrous material on the TL of silencers with three axial-direction layers (the first and third layers are void, and the second is fibrous material with filling density of 100 g/l) with $L_1 : L_2 : L_3$ of: ——, 0 : 8 : 8; -----, 1 : 8 : 7; ..., 2 : 8 : 6; —— ——, 3 : 8 : 5; —— —— ——, 4 : 8 : 4

![Graph showing Transmission Loss vs Frequency for various fibrous material locations. The graph includes a legend with the fibrous material ratios for each line segment.]
Due to the presence of void, the total amount of fibrous materials in the silencers discussed thus far in Figures 3–5 is smaller than that in an expansion chamber of the same geometry and fully filled with 100 g/l absorbent. This results from the fact that the absorbent density in the filled sections has been maintained thus far. Next, the total amount of fibrous material is kept the same, while allowing voids and uneven distribution of the fibrous material in the chamber. Figure 6 presents the results of this type of silencer, where three–layers (void, 100 g/l fibrous material, and 200 g/l fibrous material) silencers are studied. Three silencers with the following layer arrangements are examined: 100 g/l, void, 200 g/l (Figure 6(a)); void, 100 g/l, 200 g/l (Figure 6(b)); and void, 200 g/l, 100 g/l (Figure 6(c)). To retain the total amount of fibrous material the same, the volume of the void is set equal to that of the 200 g/l fibre part for all the silencers. The results of two extreme cases (one is only filled with 100 g/l fibrous material; the other one is half void and half filled with 200 g/l) are also superimposed. For silencers with the void as the middle layer (Figure 6(a)), the acoustic attenuation, in general, deteriorates with void volume. Consider, for example, the silencer of an extreme case – half void and half filled with 200 g/l \((L_1 : L_2 : L_3 = 0 : 16 : 16)\) – : TL may be reduced by up to 8 dB from mid to high frequencies. However, if the volume of void is small compared to the total volume of the chamber \((L_1 : L_2 : L_3 = 30 : 1 : 1, \text{ or } 28 : 2 : 2)\), the reduction in TL becomes nearly negligible (less than 3 dB). In addition, with the exception of the extreme case of half void and half filled with 200 g/l \((L_1 : L_2 : L_3 = 0 : 16 : 16)\), the influence of the uneven distribution is not visible at low frequencies (below 250 Hz). The TL of silencers with the mid layer as an absorbent of 100 g/l (Figure 6(b)) is similar to that of silencers with the mid layer void (Figure 6(a)). For the silencers with mid layer at 200 g/l (Figure 6(c)), the influence of uneven distribution of the absorbent may be neglected for small voids \((L_1 : L_2 : L_3 = 1 : 1 : 30, \text{ or, } 2 : 2 : 28)\) or low frequencies (below 250 Hz). The void of relatively large volume ratio \((L_1 : L_2 : L_3 = 4 : 4 : 24, \text{ or, } 8 : 8 : 16)\) results in some TL increase at mid frequencies (from 500 Hz to 2000 Hz), and some decrease at higher frequencies (above 2000 Hz).

To better quantify the effect of void volume on the acoustic performance, Table 1 summarises the peak values of TL (\(\text{TL}_{\text{peak}}\)) and the maximum reduction in TL (\(\Delta \text{TL}_{\text{max}}\)) for different voids with volumes varying from 1% to 10% (in 1% increments) that of the expansion chamber. These calculations were performed for 4 different silencers presented thus far:

- two–annular–layers (void, 100 g/l) silencer (= Figure 3)
- three–annular–layers (100 g/l, void, 200 g/l) silencer (= Figure 6(a))
- three–annular–layers (void, 100 g/l, 200 g/l) silencer (= Figure 6(b))
- three–annular–layers (void, 200 g/l, 100 g/l) silencer (= Figure 6(c)).

Numerical values clearly demonstrate that the effect of a relatively small void is nearly negligible: with 5% void fraction, maximum TL difference is confined to about 1 dB; doubling this fraction to 10% nearly doubles the maximum TL difference, though still small relative to the acoustic attenuation of the baseline configurations.
Figure 6 TL of a perforated dissipative expansion chamber with three axial-direction layers with the total amount of fibrous material being kept the same. The first, second, and third layers are (a) 100 g/l, void, 200 g/l with $L_1 : L_2 : L_3$ of: ----, 32 : 0 : 0; ------, 32 : 1 : 1; ----- 28 : 2 : 2; ---- , 24 : 4 : 4; ---, 16 : 8 : 8; +, 0 : 16 : 16; (b) void, 100 g/l, 200 g/l with $L_1 : L_2 : L_3$ of: -----, 0 : 32 : 0; ----, 1 : 30 : 1; ... , 2 : 28 : 2; ---, 4 : 24 : 4; ---, 8 : 16 : 8; +, 16 : 0 : 16 and (c) void, 200 g/l, 100 g/l with $L_1 : L_2 : L_3$ of: ----, 0 : 0 : 32; ----, 1 : 1 : 30; ... , 2 : 2 : 28; ---, 4 : 4 : 24; ---, 8 : 8 : 16; +, 16 : 16 : 0
<table>
<thead>
<tr>
<th>Void ratio</th>
<th>Two-layers Void, 100 g/l</th>
<th>Two-layers Void, 200 g/l</th>
<th>Three-layers Void, 100 g/l</th>
<th>Three-layers Void, 200 g/l</th>
<th>Maximum reduction in TL (ΔTL_{max}) (dB) Void, 100 g/l</th>
<th>Maximum reduction in TL (ΔTL_{max}) (dB) Void, 100 g/l</th>
<th>Maximum reduction in TL (ΔTL_{max}) (dB) Void, 100 g/l</th>
<th>Maximum reduction in TL (ΔTL_{max}) (dB) Void, 200 g/l</th>
</tr>
</thead>
<tbody>
<tr>
<td>No void</td>
<td>38.1</td>
<td>-0.1</td>
<td>38.1</td>
<td>-0.1</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.2</td>
</tr>
<tr>
<td>1 %</td>
<td>38.0</td>
<td>-0.2</td>
<td>38.0</td>
<td>-0.2</td>
<td>-0.4</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>2 %</td>
<td>37.9</td>
<td>-0.3</td>
<td>37.8</td>
<td>-0.4</td>
<td>-0.6</td>
<td>-0.4</td>
<td>-0.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>3 %</td>
<td>37.7</td>
<td>-0.5</td>
<td>37.7</td>
<td>-0.5</td>
<td>-0.9</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.6</td>
</tr>
<tr>
<td>4 %</td>
<td>37.6</td>
<td>-0.6</td>
<td>37.5</td>
<td>-0.7</td>
<td>-1.1</td>
<td>-0.7</td>
<td>-0.7</td>
<td>-0.7</td>
</tr>
<tr>
<td>5 %</td>
<td>37.5</td>
<td>-0.7</td>
<td>37.3</td>
<td>-0.9</td>
<td>-1.3</td>
<td>-0.6</td>
<td>-1.0</td>
<td>-0.8</td>
</tr>
<tr>
<td>6 %</td>
<td>37.4</td>
<td>-0.8</td>
<td>37.1</td>
<td>-1.1</td>
<td>-1.5</td>
<td>-0.6</td>
<td>-1.2</td>
<td>-0.9</td>
</tr>
<tr>
<td>7 %</td>
<td>37.3</td>
<td>-0.9</td>
<td>36.9</td>
<td>-1.3</td>
<td>-1.7</td>
<td>-0.7</td>
<td>-1.4</td>
<td>-1.0</td>
</tr>
<tr>
<td>8 %</td>
<td>37.1</td>
<td>-1.1</td>
<td>36.7</td>
<td>-1.5</td>
<td>-1.9</td>
<td>-0.9</td>
<td>-1.7</td>
<td>-1.0</td>
</tr>
<tr>
<td>9 %</td>
<td>37.0</td>
<td>-1.2</td>
<td>36.5</td>
<td>-1.7</td>
<td>-2.1</td>
<td>-1.1</td>
<td>-1.9</td>
<td>-1.1</td>
</tr>
<tr>
<td>10 %</td>
<td>37.2</td>
<td>-1.3</td>
<td>36.2</td>
<td>-1.8</td>
<td>-2.4</td>
<td>-1.3</td>
<td>-2.1</td>
<td>-1.5</td>
</tr>
</tbody>
</table>
4 Conclusions

The effect of voids on the attenuation performance of perforated dissipative expansion chambers has been investigated, by modelling such segments as empty annular layers in the expansion chamber. The TL has been determined using a two-dimensional, closed-form analytical approach through the application of boundary conditions of acoustic pressure and velocity continuities. The accuracy of this analytical approach has been verified by the reasonable agreement of the analytical predictions with both the numerical results and experiments for one specific configuration. The effects of void length and location, and unevenly distributed fibrous material on the acoustic attenuation of the perforated dissipative expansion chamber have been illustrated. The impact of a relatively small void (less than 10% of the total volume) was found to be nearly negligible (with the maximum reduction in TL within 2 dB or less relative to 38.2 dB for the baseline configuration considered here), regardless of its location, and the associated uneven filling. At low frequencies (below 250 Hz), the effect of void on the acoustic attenuation was also observed to be negligible.

References


Lee, I.J. (2005) Acoustic Characteristics of Perforated Dissipative and Hybrid Silencers, PhD Dissertation, The Ohio State University, Columbus, OH.


**Appendix A: Acoustic properties of fibre material**

The acoustic properties of the fibrous material are characterised by the complex-valued characteristic impedance \( \bar{Z} = \bar{\rho} \bar{c} \) and wave number \( \bar{k} = 2\pi f / \bar{c} \), which are experimentally determined and given (Lee, 2005), for 100 g/l, by,

\[
\frac{\bar{Z}}{Z_0} = [1 + 33.20(f)^{-0.7523}] + j[28.32(f)^{-0.6512}],
\]

(A1)

\[
\frac{\bar{k}}{k_0} = [1 + 39.2(f)^{-0.6841}] + j[38.39(f)^{-0.6285}];
\]

(A2)

and, for 200 g/l,

\[
\frac{\bar{Z}}{Z_0} = [1 + 25.69(f)^{-0.5523}] + j[71.97(f)^{-0.7072}]
\]

(A3)

\[
\frac{\bar{k}}{k_0} = [1 + 56.03(f)^{-0.6304}] + j[62.05(f)^{-0.5980}]
\]

(A4)

where \( Z_0 = \rho_0 c_0 \) is the characteristic impedance of the air.
Appendix B: Acoustic impedance of perforates

The non-dimensionalised perforate acoustic impedance $\tilde{\zeta}$ relates the acoustic pressure in the inner duct and outer chamber through the interface. An empirical expression for the acoustic impedance of perforate holes facing air is experimentally determined and expressed as (Lee, 2005)

$$\tilde{\zeta} = [0.005101 + jk_o(t_w + 0.4409d_h)]/\phi.$$  \hspace{1cm} (B1)

In the presence of fibre (for perforations facing absorbing material), equation (B1) has been replaced by (Lee, 2005)

$$\tilde{\zeta} = [0.02996 + jk_o(t_w + 0.6471d_h)]/\phi, \hspace{0.5cm} \text{for } 100 \text{ g/l},$$  \hspace{1cm} (B2)

$$\tilde{\zeta} = [0.04575 + jk_o(t_w + 0.7412d_h)]/\phi, \hspace{0.5cm} \text{for } 200 \text{ g/l}.$$  \hspace{1cm} (B3)